The geometric principles of string instrument-making in Brescia

François Denis

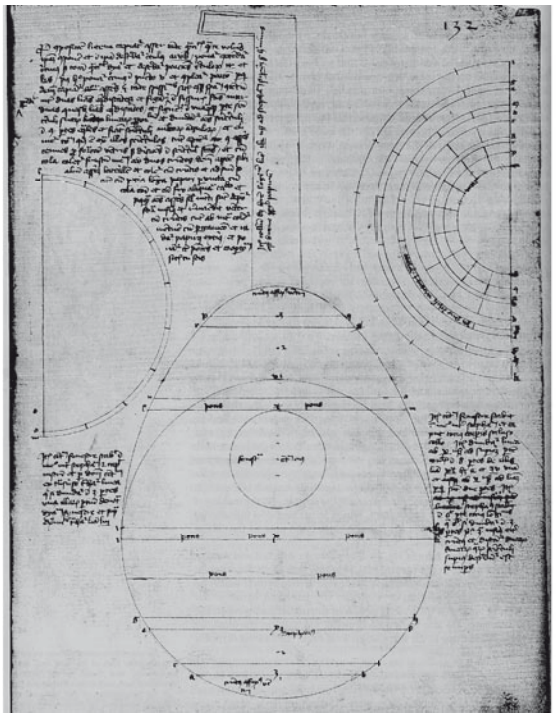
We know how craftsmen of former ages used geometry from the evidence of various practical handbooks and treatises, one of which, written by Henri Arnaud de Zwolle in the 15th century, is partly devoted to the lute.[1] Detailed study of these documents bears witness to the ancient art of drawing with ruler and compass. Long nurtured by the oral traditions of craftsmanship, the skill fell into disuse during the 17th century.

Picture of a lute in Spain c. 1260. In Islamic culture it was the instrument of the elite and was the first to benefit from elaborate construction techniques.

To reveal these now vanished processes, it is sufficient to compare the measurements of an instrument's different dimensions.[2] We should remember, however, that such relations are legion and their limits often ill-defined. Consequently, interpreting them is a much more complex business than might be supposed from the primary evidence of their analysis. Too much data can conceal the bigger picture; a selection must be made, informed by familiarity with the history of science, technology and the arts.

To complicate matters further, the notions of measurement and geometry are not the same now as they were in the Renaissance. An objective account of the techniques for ruler-and-compass drawing used in those bygone days therefore requires some prior explanation.

Once these precautions have been taken, the instruments' dimensions are like a forgotten alphabet whose surprising elegance questions the whole idea of progress in instrument-making.



The only plan of a musical instrument to have survived from the 15th century, we owe this drawing of a lute, together with a description of its measurements and the principles for building it, to the astronomer Henri Arnaut de Zwolle. It contains information essential to an understanding of all instrument-making until the 17th century.

FRAMEWORKS

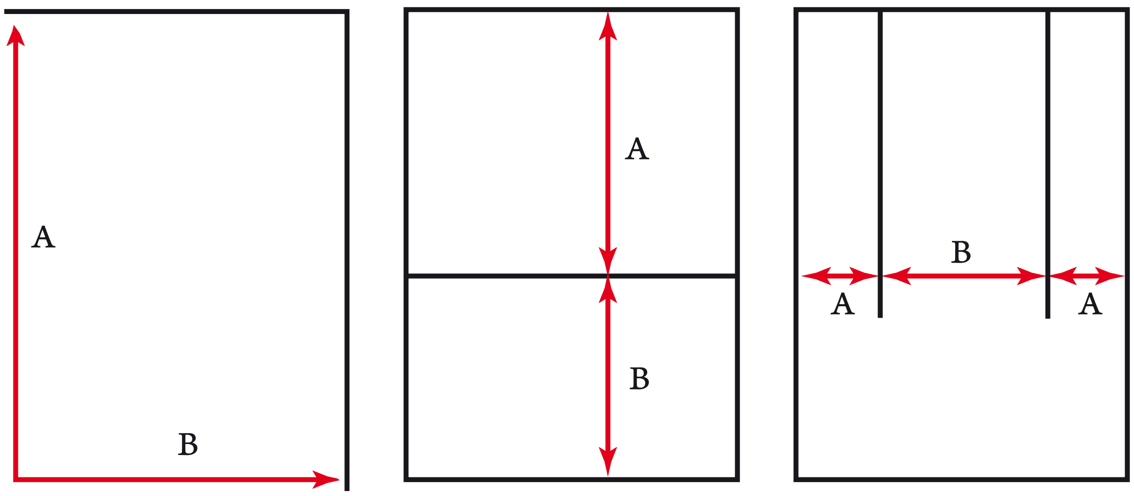
Three ages of the history of forms

During the geometric period compass in pictures commonly symbolises reasoning and measuring.
(Melancholy - Dürer)

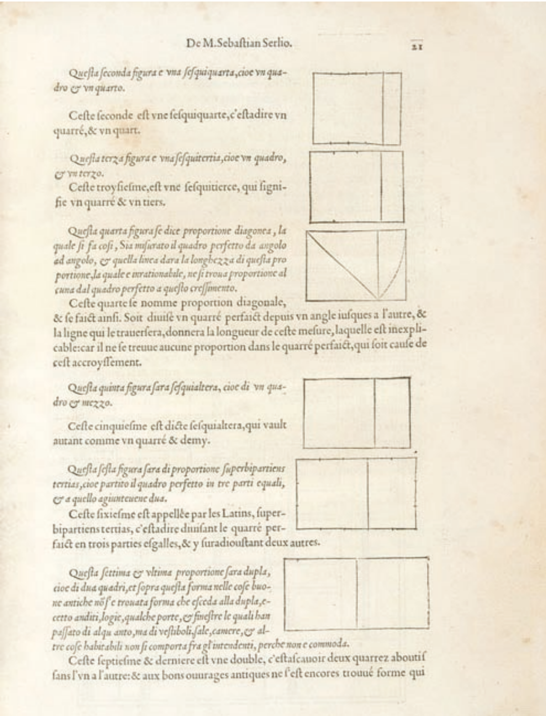
Three periods may be distinguished in the history of the forms of bowed and plucked string instruments from the Middle Ages to the present day. From the 13th to the mid-17th century, craftsmen made their models with ruler and compass according to the "art of measurement".[3] This first, so-called "geometric" period was followed by a transitional period lasting about a hundred years, corresponding to the golden age of Italian instrument-making. Despite its glorious association, it was during this "post-geometric" period that the techniques of compass drawing started to decline with the spread of increasingly empirical methods.[4] The transition ended in decadence as the ancient knowledge and skills vanished completely. It was followed by the "romantic" period, which extends from the mid-18th century to the present day. This period, in which the creation of forms depends entirely on the outlines of existing instruments, also saw the emergence of the mythical (and rival) figures who would henceforth inform both the imagination and the organisation of the profession.

The basics

At the time when compasses were still widely used, designing an object consisted in pragmatically defining the limits "of the whole and its parts".[5] The outline of a bowed or plucked string instrument was inscribed within an organised surface called a framework. This organisation considered only the ratios of adjacent dimensions[6] of length and width.



Each of these relations corresponds to a single manipulation of the ruler or compass and forms part of a logical sequence. All the dimensions therefore depend on a clearly defined order of operations. It is these various procedures that the Ancients called "measuring".



De M.Sebaſtian Serlio. 21

Queſta ſeconda figura e vna ſeſquiquarta, cioe vn quadro & vn quarto.

Ceſte ſeconde eſt vne ſeſquiquarte, c'eſtadire vn quarré, & vn quart.

Queſta terza figura e vna ſeſquitertia, cioe vn quadro, & vn terzo.

Ceſte troyſieſme, eſt vne ſeſquitierce, qui ſignifie vn quarré & vn tiers.

Queſta quarta figura ſe dice proportione diagonea, la quale ſi fa coſi, Sia miſurato il quadro perfetto da angolo ad angolo, & quella linea dara la longhezza di queſta proportione, la quale e irrationabale, ne ſi troua proportione alcuna dal quadro perfetto a queſto creſſimento.

Ceſte quarte ſe nomme proportion diagonale, & ſe faict ainſi. Soit diuiſé vn quarré perfaict depuis vn angle iuſques a l'autre, & la ligne qui le trauerſera, donnera la longueur de ceſte meſure, laquelle eſt inexplicable: car il ne ſe treuue aucune proportion dans le quarré perfaict, qui ſoit cauſe de ceſt accroyſſement.

Queſta quinta figura ſara ſeſquialtera, cioe di vn quadro & mezzo.

Ceſte cinquieſme eſt dicte ſeſquialtera, qui vault autant comme vn quarré & demy.

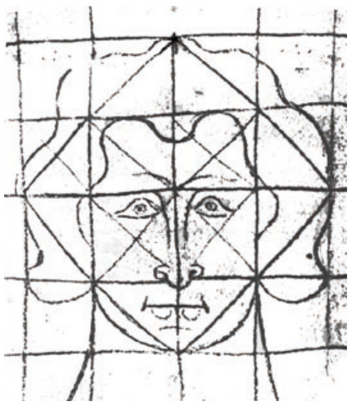
Queſta ſeſta figura ſara di proportione ſuperbipartiens tertias, cioe partito il quadro perfetto in tre parti equali, & a quello agiunteuene dua.

Ceſte ſixieſme eſt appellée par les Latins, ſuperbipartiens tertias, c'eſtadire diuiſant le quarré perfaict en trois parties eſgalles, & y ſuradiouſtant deux autres.

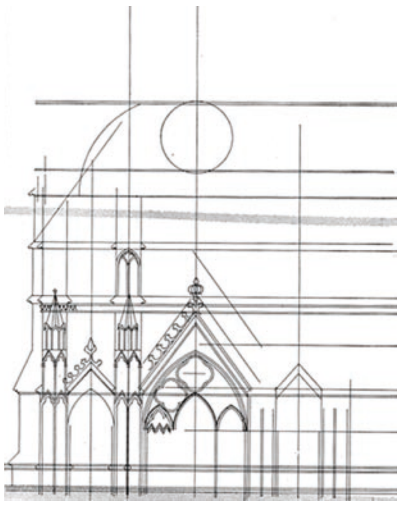
Queſta ſettima & vltima proportione ſara dupla, cioe di dua quadri, et ſopra queſta forma nelle coſe buone antiche nō ſ'e trouata forma che eſceda alla dupla, eccetto anditi, logie, qualche porte, & fineſtre le quali han paſſato di alqu anto, ma di veſtiboli, ſale, camere, & altre coſe habitabili non ſi comporta fra gl'intendenti, perche non e commoda.

Ceſte ſeptieſme & derniere eſt vne double, c'eſtaſcauoir deux quarrez aboutiſſans l'vn a l'autre: & aux bons ouurages antiques ne ſ'eſt encores trouué forme qui

The different formats of a form have been allocated between the square and the double square according to the measurement of the added part.
After S. Serlio



The quadrature of the square represents the archetypal vertical and horizontal division of a surface. It is also the basis for proportional symmetry, a notion which assumes various meanings that in turn provided the basis for a balanced composition of form.
The sketch-book of Villars de Honnecourt, 13th century.

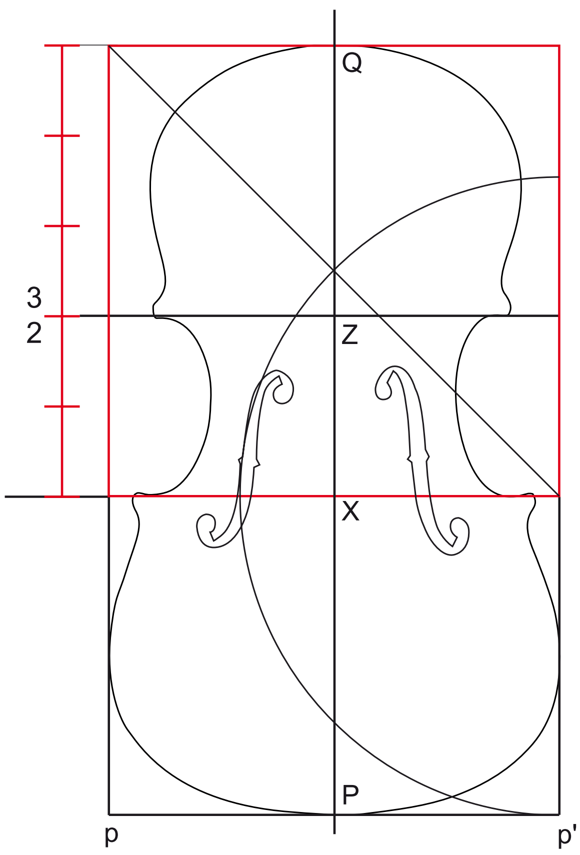


The Rheims palimpsest is one of the few surviving plans of a medieval cathedral. It should be read from right to left and from top to bottom. It shows the different stages of design, beginning with the most general relations and ending with details of the decoration.
Rheims, City Library.

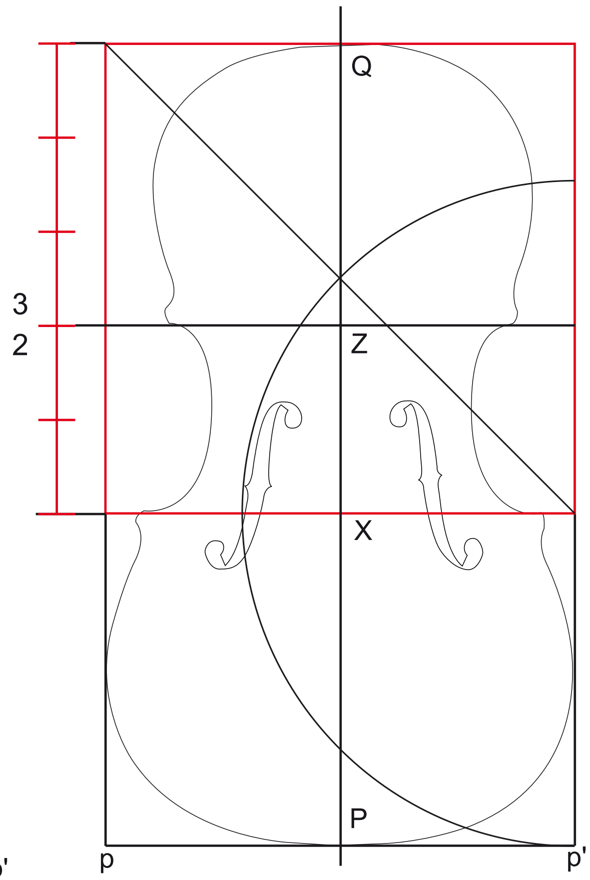
Violins [7]

Surface and vertical divisions

The proportions of Brescian instruments apply to the dimensions of the outline of the edges, whereas in Cremona the significant dimensions are those within the ribs. As we shall see below (Notes on making process), this difference is probably due to the construction method. The violins were made in two sizes according to an identical geometrical model. Violins N14 and N11 are good illustrations of this principle. The quotients of the different ratios of length and width and of the lengths of the lower, middle and upper parts (Table 2 in the annex) produce a definition of so-called "harmonic" surfaces.[8]



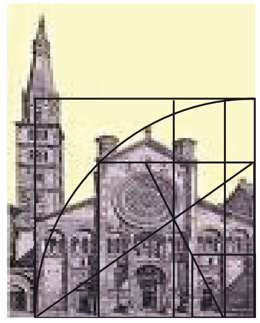
The construction of violin N14 starts with a square with sides equal to XQ divided into five parts. Rotating the half-diagonal downwards defines the rectangle that frames the form and the dimension of XP, the height of the lower part of the form. The midpart XZ measures two parts of the square (i.e. a ratio of 2 : 3 between the midpart XZ and the upper part ZQ).



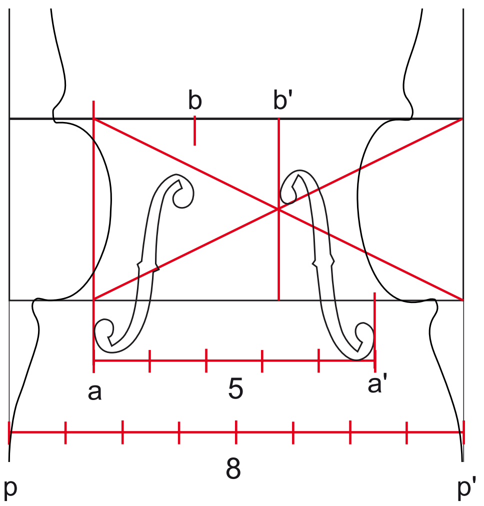
Though longer by 8mm and wider by 3.5mm, the proportions of violin N11 are identical to those of the previous instrument.

Horizontal divisions - Placing and measuring the f-holes

The relative dimensions of the width are determined according to a geometrical procedure similar to the one used by the Amati at the same period. The maximum width of the lower part pp' is the starting dimension, from which the maximum spacing between the f-holes aa' is derived. It is therefore always relative to a division of pp'. The minimum spacing between the f-holes bb' is equal to half the difference between pp' and bb'. It is determined by the simple construction shown in the diagram below. The logic of this ordering of widths is apparent from a study of the ratios given in Table 3 in the annex.



The methods for determining the widths and the openings of musical instruments were related to those used in architecture at the same time.

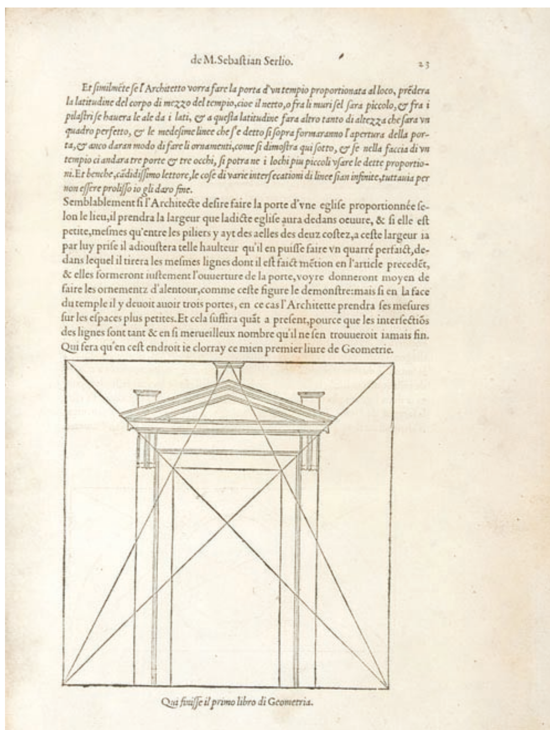


With violin N14, the width is divided into 8 parts, 5 parts being given to aa', the maximum spacing between the f-holes, and 1.5 parts to bb', the minimum spacing between the f-holes.

The relative dimensions of the f-holes are taken from Table 4 in the annex. The geometrical method is borrowed from the architecture of façades. The openings are placed on a plane according firstly to their two principal dimensions, i.e. the height and width of the surfaces on which they are inscribed, and not the measurements made from the centres of the holes. Thus, the two f-holes of violin N11 are situated in a 3 by 5 rectangle, while those of violin N14 are situated in a 7 by 10 rectangle.

It is likely that each f-hole was initially inscribed in a given surface, but the scale of the asymmetries restricts any interpretation on this point.

More generally, these relations deriving from the initial division into 5 or 8 of the instrument's greatest width appear to owe much to the proportional relations attaching to the Fibonacci sequence. [9]



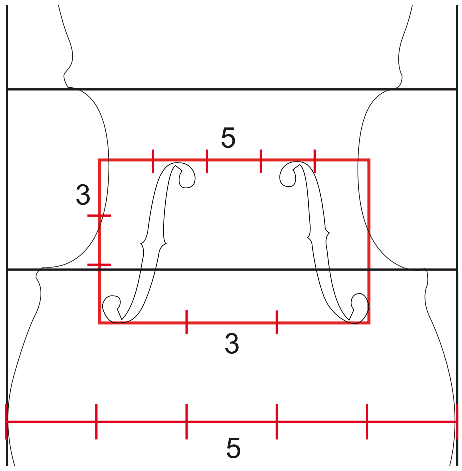
de M. Sebastian Serlio. 23

Et similmẽte se l'Architetto vorra fare la porta d'vn tempio proportionata al loco, prẽdera la latitudine del corpo di mezzo del tempio, cioe il netto, o fra li muri s'el sara piccolo, & fra i pilastri se hauera le ale da i lati, & a questa latitudine fara altro tanto di altezza che sara vn quadro perfetto, & le medesime linee che s'e detto si sopra formaranno l'apertura della porta, & anco daran modo di fare li ornamenti, come si dimostra qui sotto, & se nella faccia di vn tempio ci andara tre porte & tre occhi, si potra ne i lochi piu piccoli vsare le dette proportioni. Et benche, cãdidissimo lettore, le cose di varie intersecationi di linee sian infinite, tuttauia per non essere prolisso io gli daro fine.

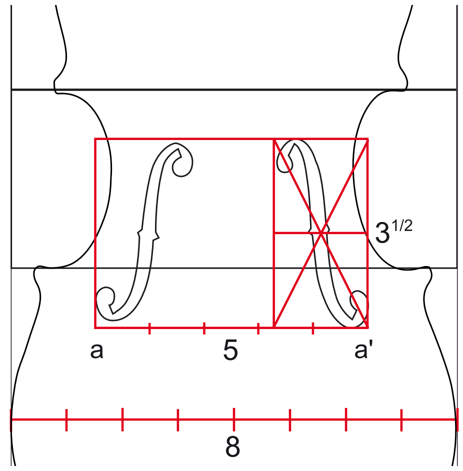
Semblablement si l'Architecte desire faire la porte d'vne eglise proportionnée selon le lieu, il prendra la largeur que ladicte eglise aura dedans oeuure, & si elle est petite, mesmes qu'entre les piliers y ayt des aelles des deux costez, a ceste largeur ia par luy prise il adioustera telle haulteur qu'il en puisse faire vn quarré perfaict, dedans lequel il tirera les mesmes lignes dont il est faict mẽtion en l'article precedẽt, & elles formeront iustement l'ouuerture de la porte, voyre donneront moyen de faire les ornementz d'alentour, comme ceste figure le demonstre: mais si en la face du temple il y deuoit auoir trois portes, en ce cas l'Architette prendra ses mesures sur les espaces plus petites. Et cela suffira quãt a present, pource que les intersectiõs des lignes sont tant & en si merueilleux nombre qu'il ne s'en trouueroit iamais fin. Qui sera qu'en cest endroit ie clorray ce mien premier liure de Geometrie.

Qui finisse il primo libro di Geometria.

Using a proportional section to measure a door. (After S. Serlio).

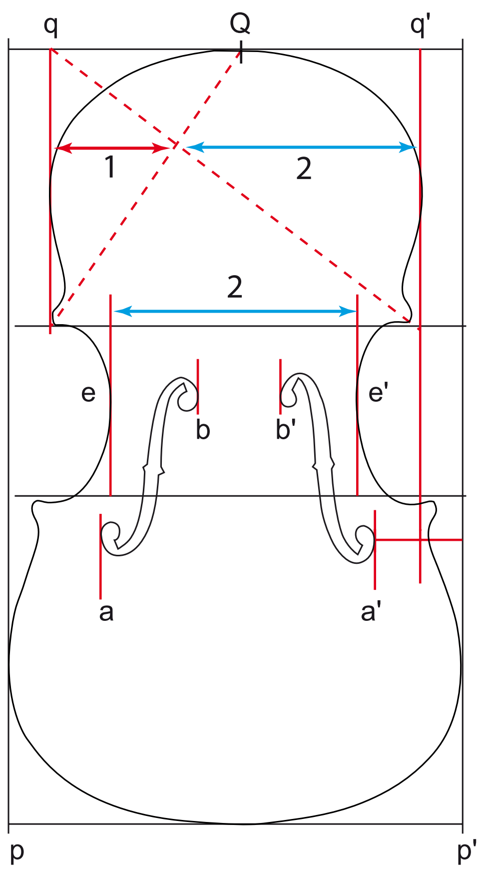


The 3 to 5 proportion between pp' and aa' is found again in the measurement of the main dimensions of the f-holes, since they are inscribed in a rectangle with the same 3 by 5 relation.



The height of the f-holes is measured by 3.5 of these parts, such that the two f-holes are inscribed in a 5 by 3.5 (10 by 7) rectangle. As before, each f-hole is itself inscribed in a double square.

The division of widths continues in an elegant manner. The maximum width of the upper part qq' derives from a division into 2 of the space between the vertical lines passing through p and a (and symmetrically for the space between a' and p'), lastly ee', the minimum width of the midpart, is equal to two-thirds of qq'.

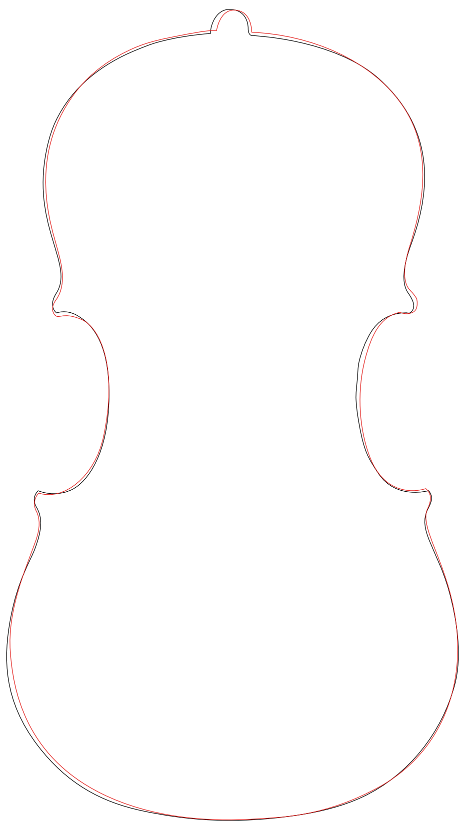


Geometrical construction of qq', the maximum width of the upper part, and ee', the minimum width of the midpart (violin N14). In accordance with the principles of symmetry, still perceptible even though the making process is often approximate, bb' (the minimum spacing between the f-holes) is a measurement close to the difference between pp' (the maximum width of the lower part) and qq' (the maximum width of the upper part). bb' is also equal to half the difference between pp' and aa' (the maximum spacing between the f-holes). This ordering of the framework, identical to that used by the Amati, may be regarded as one of the oldest of the violin family.

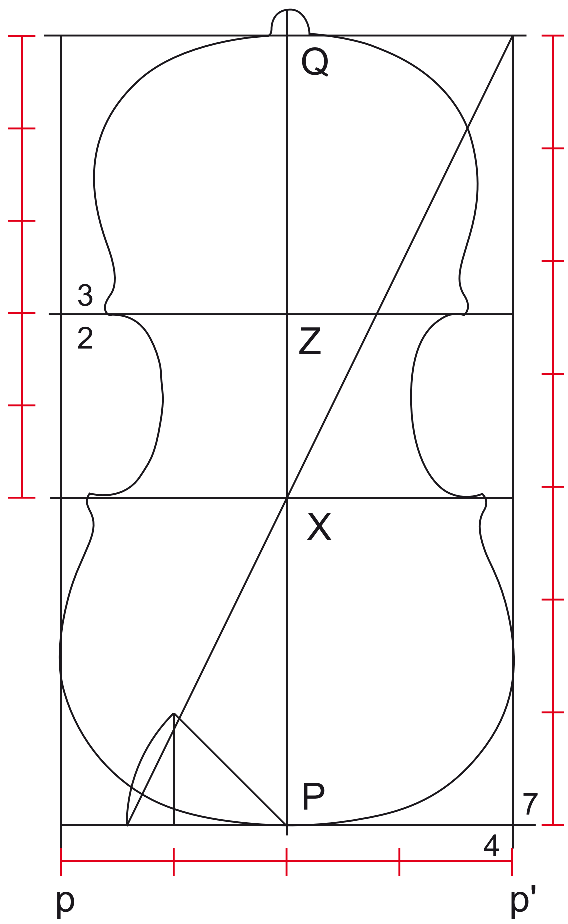
Two violas

Violas N5 and N6 of this exhibition are two large instruments which have not been recut, a relatively rare occurrence. Superposing the two outlines shows the similarity of their model.

Surface and vertical divisions

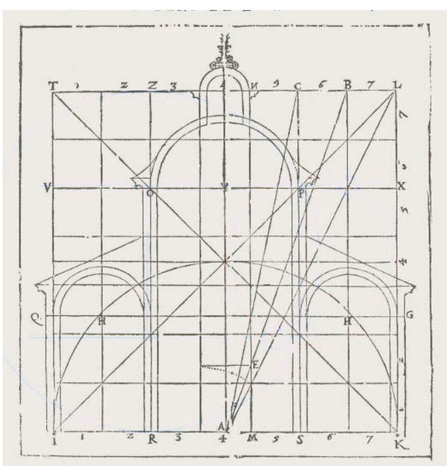


Comparison of the outlines of violas N5 and N6



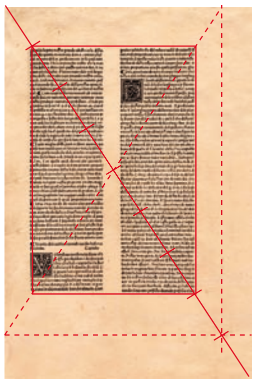
The outline of viola N6 is inscribed in a 7 by 4 rectangle (table 5 in the annex). The remarkable feature of this framework is the placement of X (XP : XQ = 0.414). Diagonals are used to construct the figure. The point Z is situated by dividing XQ into 5.

Construction by the diagonal of the square. (After S. Serlio)

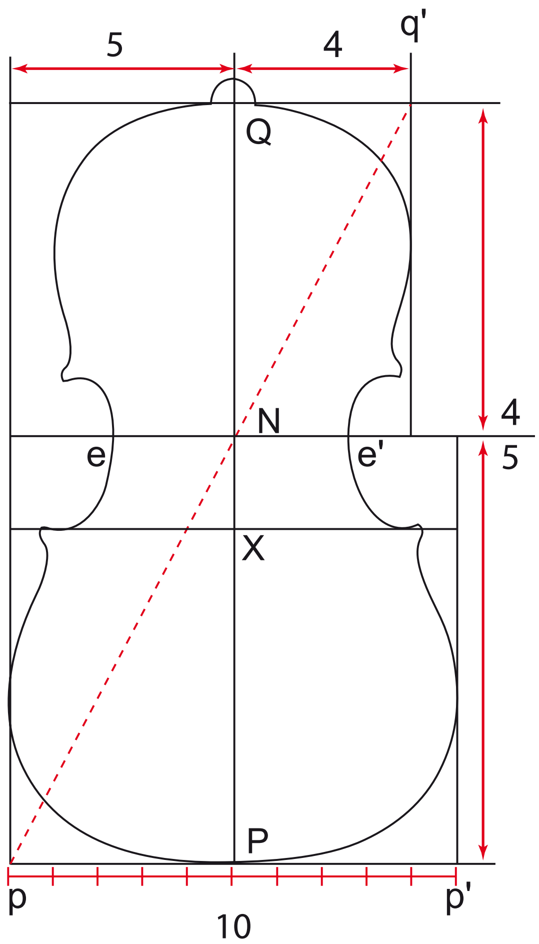


Example of a drawing process similar to that of the viola applied to the construction of facades. (After Francesco di Giorgio Martini)

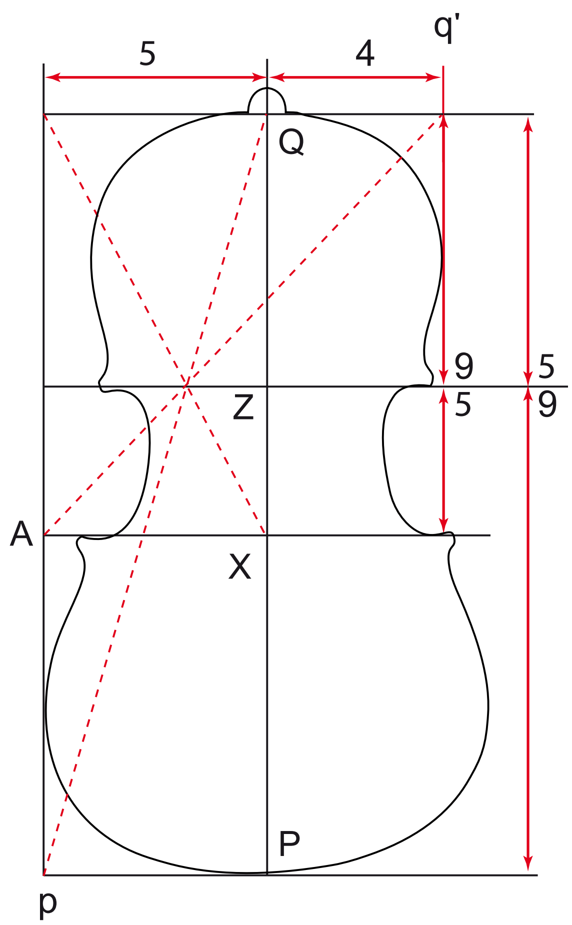
The outlines of violas N17 and N18 (and N10) are also inscribed in harmonic rectangles (see Table 7 in the annex). They form a group whose similarity derives from the symmetrical placement of the points X and N, respectively the limit of the lower corners and the minimum width of the midpart (see Table 8 in the annex).



It is known that diagonals were used to set type in the earliest days of printing. This Gutenberg bible is also based on a division of the height into nine parts. (After Adolf Wild, Cahiers de Gutenberg no. 22, 09/1995)



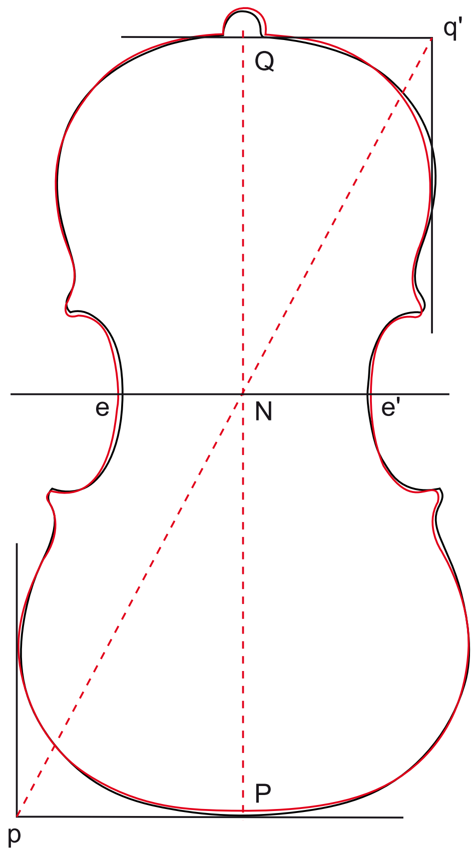
The width pp' being divided into 10 parts, construct a harmonic rectangle (i.e. PQ = 17 parts), q' is placed one part in, Qq' is half the width of the upper part. Draw a diagonal joining p to q' (at the level of the horizontal lines passing through P and Q) to place N, the midpoint of the minimum width of the midpart. As a result, the length is divided into 9 parts at the level of the line PQ and the point N is situated 4 of those parts below Q. Place X below N such that PX equals NQ.



A diagonal drawn from p to Q or from A to q' cuts the diagonal through X at the level of the horizontal line through Z, the limit of the upper corners (with XZ is to ZQ as 5 is to 9).

Commentary

One interesting aesthetic consequence of this geometric method is that ee', the narrowest width of the midpart, descends relative to P when the width of the upper part qq' increases relative to that of the lower part pp'. Consequently, the overall aspect of the form is directly linked to the relative widths of the upper and lower parts.

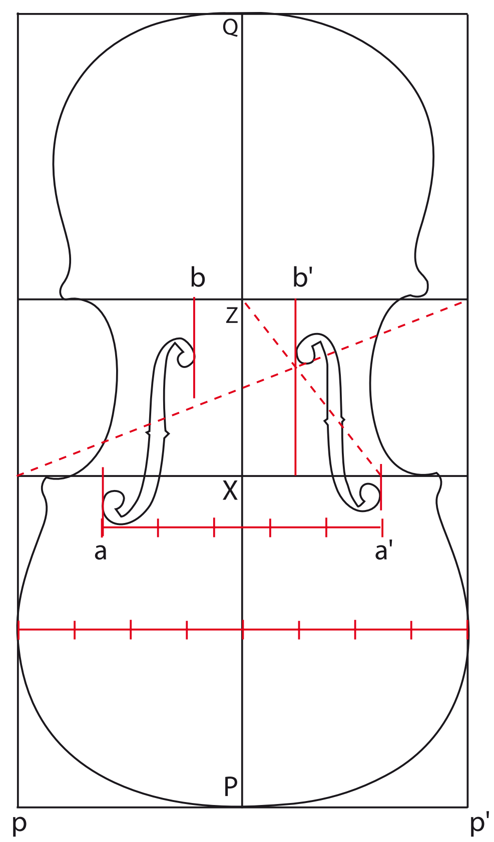


Example of the use of diagonals to apply proportion to figures. (After Villard de Honnecourt, 13th century)

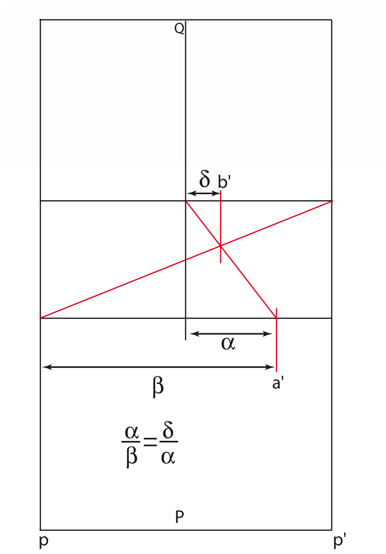
Superposing the outline of viola N6 (in black) with that of the Charles IX tenor viola (Ashmolean Museum) reduced to the same length (PQ) shows a similarity of contour and proportionality. The great width of the upper part relative to the lower part descends the point N towards P. Consequently, the upper part of the midpart curve is more open.

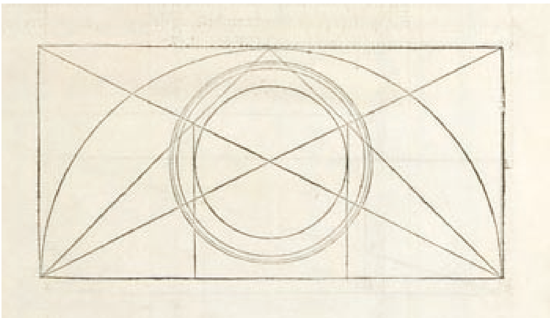
Horizontal divisions - Placing and measuring the f-holes

The same geometric processes already described for violins are used here. The organisation starts with a division of the maximum width of the lower part pp', on which the measurements of the f-holes depend, and continues with the relative widths of the upper and midpart.



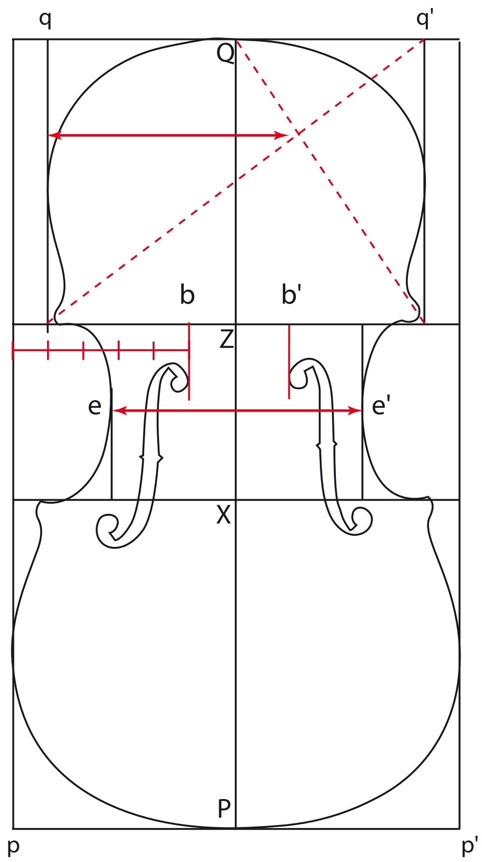
The widths of violas N5 and N6 (table 6 in annex) are calculated from a division into 8 parts of the width pp'. The maximum spacing of the f-holes aa' is equal to 5 of these parts. The ratio of 5 to 8 echoes the famous Fibonacci sequence, which is elegantly associated here with the homothetic relations of the triangles.





This recommendation for placing the opening of a façade (after Serlio) is an illustration of the principles applied to the construction of musical instruments. The surface (here the double square) is divided by a proportional section (here the arithmetic section).

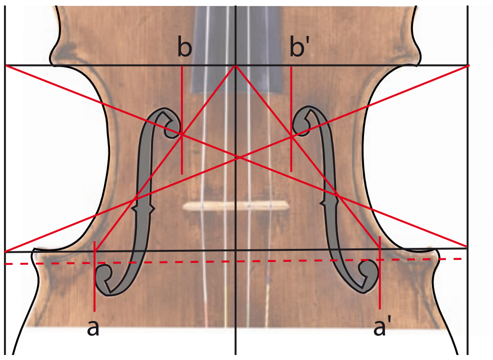
The horizontal organisation continues with a division of the remaining adjacent and symmetrical parts. Only the spaces between the verticals of the previously defined widths meet these two conditions. Various possibilities exist, including the one shown below for the two violas N5 and N6.

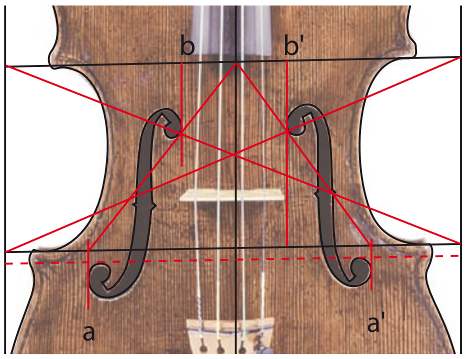


The space between the vertical lines passing through b and p is divided into 5 parts: the space between b and q is equal to 4 of those parts and qq' is the maximum width of the upper part. The space between the vertical lines passing through q and q' is divided into 3 parts. ee', the minimum width of the midpart, is equal to 2 of those parts.

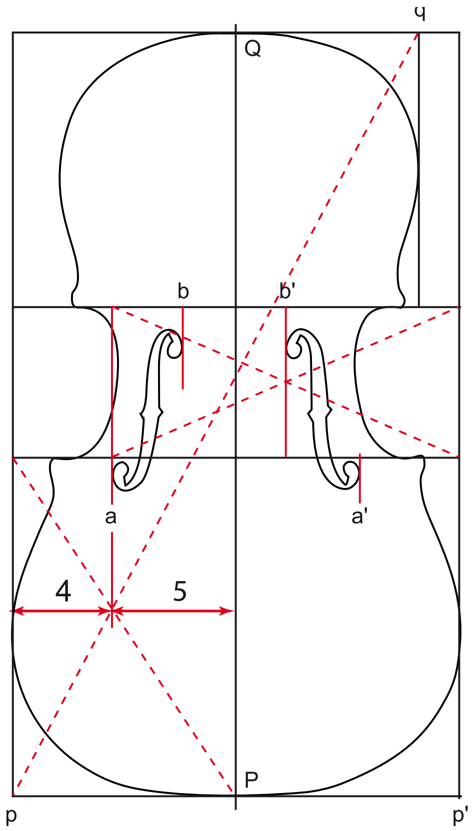
These symmetrical processes (in the proportional sense of the term) could give a paradoxical explanation of the troubling setting in f-holes to be found on many instruments of the Brescia school.

The variable orientation and length of the f-holes seem to depend on the quality of the parallelism between the lines connecting the corners.

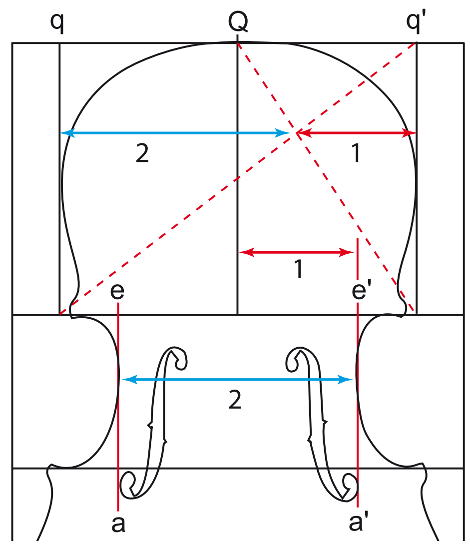




The model for the organisation of widths of the N17 and N18 violas is deduced from the relations given in Table 9 in the annex.



The maximum spacing of the f-holes aa' is to the maximum width of the lower part pp' as 5 is to 9. This ratio echoes the similar divisions of the length and corresponds to the geometrical process illustrated opposite.



As previously, ee', the minimum width of the midpart of these two instruments, is equal to two-thirds of qq', the maximum width of the upper part. Note that with viola N18, the maximum spacing of the f-holes aa' is also equal to ee', the minimum width of the midpart.

Commentary

The latter two instruments, remarkably constructed on the interval of the major third (4 to 5), seem to pay visual tribute to mean-tone. They provide evidence that instrument-makers at the time were keen to associate harmony of form with that of music.

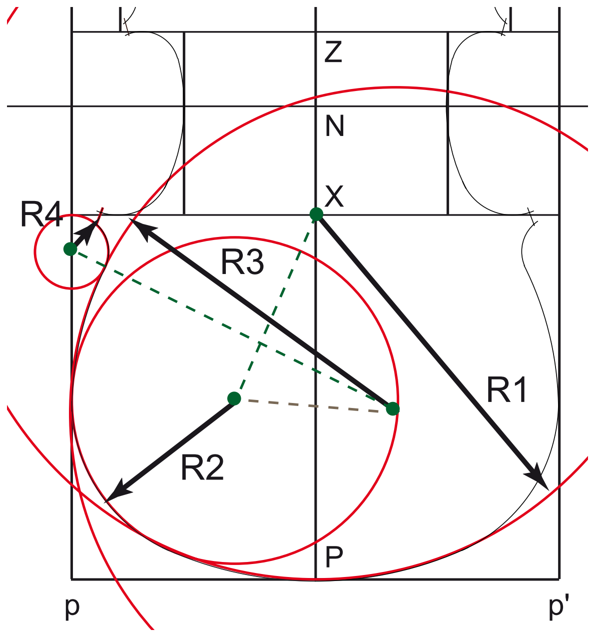
Drawing outlines

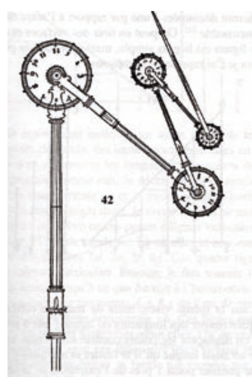
A reminder of techniques for drawing with a compass

The outlines of instruments (except for the lute, which uses the ellipse) consist of a succession of circles. The radii of these circles correspond to the spaces between the divisions marked on the axes. The limits of the drawing are set by the framework.

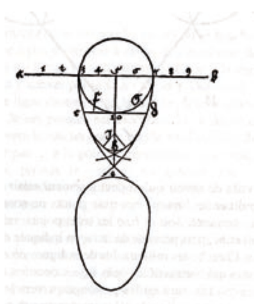
The radii form a set of articulated segments that operate like the members of a body. In this sense, the line is the trajectory of a point and drawing the outline is a geometry of movement.[11]

The radii of the arcs (R1) that draw the top and bottom blocks have the points X, N or Z as their centre. The radii of the arcs (R2) that extend those of the blocks measure half the height XP (or ZQ for the upper part) or a third of the width (i.e. 1/3 of qq' or 1/3 of pp'). The radius of arc R3 tangent to R2 that rises towards the corner is double that of R2. The dotted segments are the hidden radii that govern the construction.

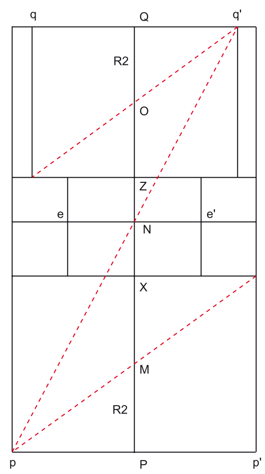




Dürer devised a way of drawing any curve using an articulated system that mimics the working of a human arm. The construction of a violin outline uses geometrical principles identical to those applied by this peculiar device.



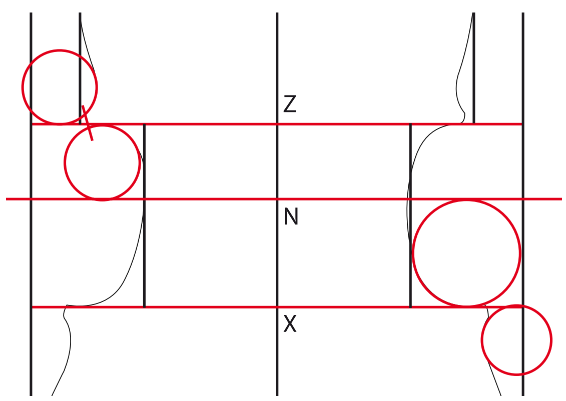
Dürer uses the ovum to illustrate the principle of drawing arcs. The figure is constructed from the divisions of an orthonormal basis.



As when drawing an ovum, the measurements of the arc radii that generate the outline of a violin will be determinated by the axis divisions.

Another frequent feature of Brescian instruments is the way in which the radii of the corner arcs are tangent to the horizontal line passing through X or Z.

This information is sufficient to draw the outline. From the following examples, we can see the recurring patterns at the origin of a Brescian style.

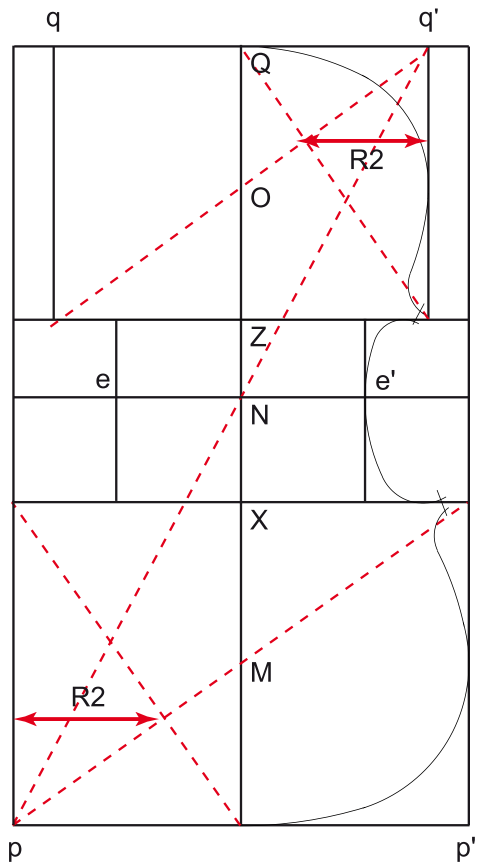


The radii of the arcs that draw the outlines of the corners and the midpart are derived from divisions of XZ.

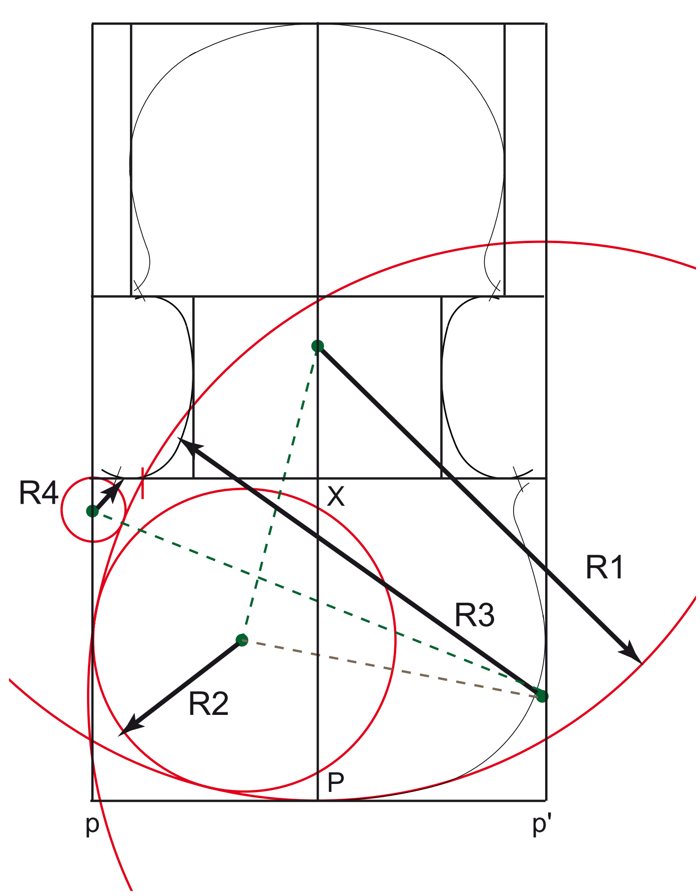
Examples of outlines

Violin N14

The principal dimensions having been established as indicated, it is remarkable that the approximate symmetry of these instruments fails to mask the solidity of the geometric principles that underlie the aesthetics of their lines. Let us remember that the process involves drawing an outline, not a mould. This nuance has perceptible repercussions on the form itself, while also simplifying certain aspects of the drawing technique. [12]

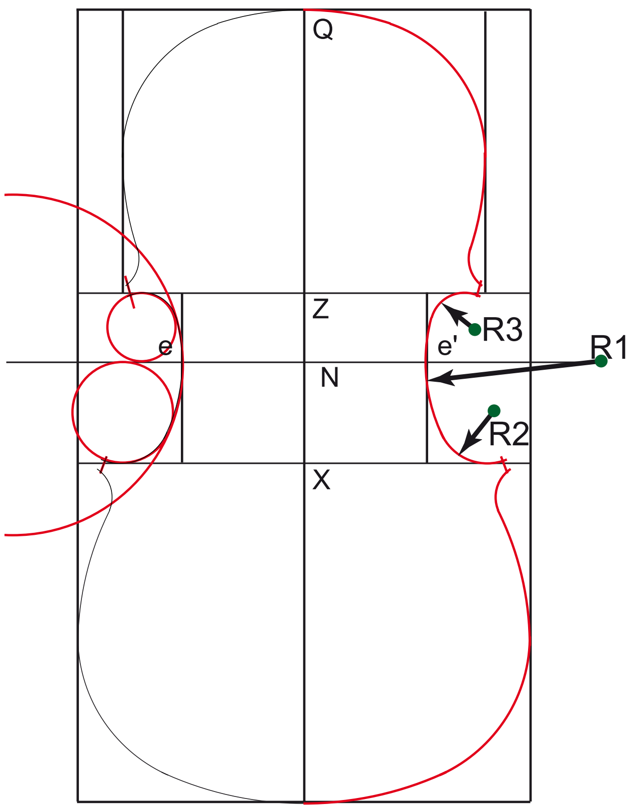


Diagonals are used to place N, the midpoint of the minimum width of the midpart, and to determine the measurements of the radii R2 (here one-third of pp' and qq') tangent to the two maximum widths. This appears to have been common practice in Brescia.

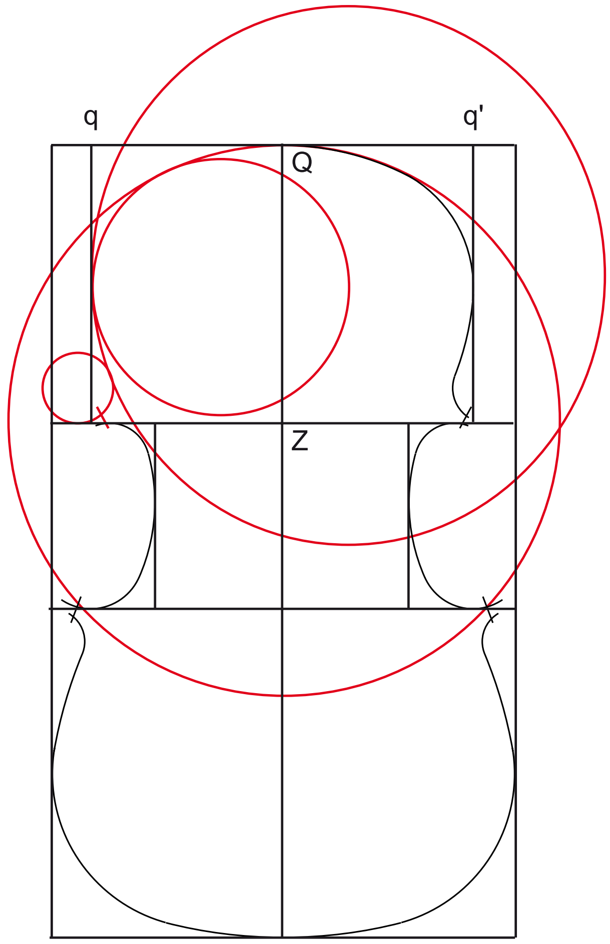


The lower part is drawn by four arcs with the following radii:

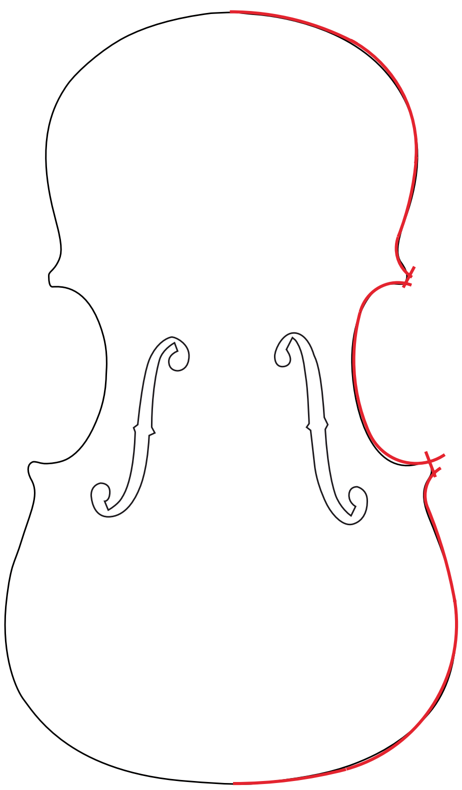
***R1** = pp'*
***R2** = 1/3 of pp'*
***R3** = pp'*
***R4** = 1/5 of XP*



The midpart is drawn by three arcs with the following radii:
R1 *= XZ* ***R2*** *= 1/2 of XZ* ***R3*** *= 1/2 of ZN*

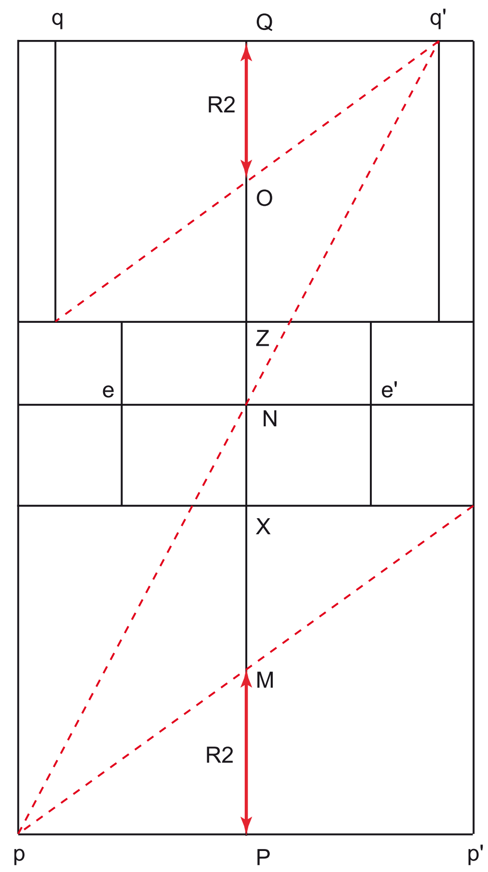


The upper part is drawn by four arcs with the following radii:
R1 *= XQ* ***R2*** *= 1/3 of qq* ***R3*** *= 2/3 of qq*
R4 *= 1/4 of ZQ*

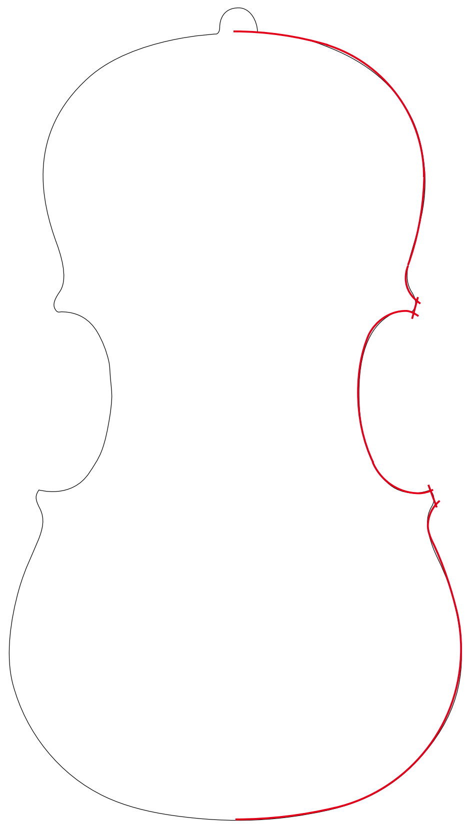


Outline of violin N14 superimposed on its geometrical reconstruction (in red).

Viola N6



The way the arcs are drawn is broadly the same for violas. Again, diagonals are used to determine the radii R2 and the point N. In the case of instrument N6, the radii R2 are equal to half PX and ZQ.



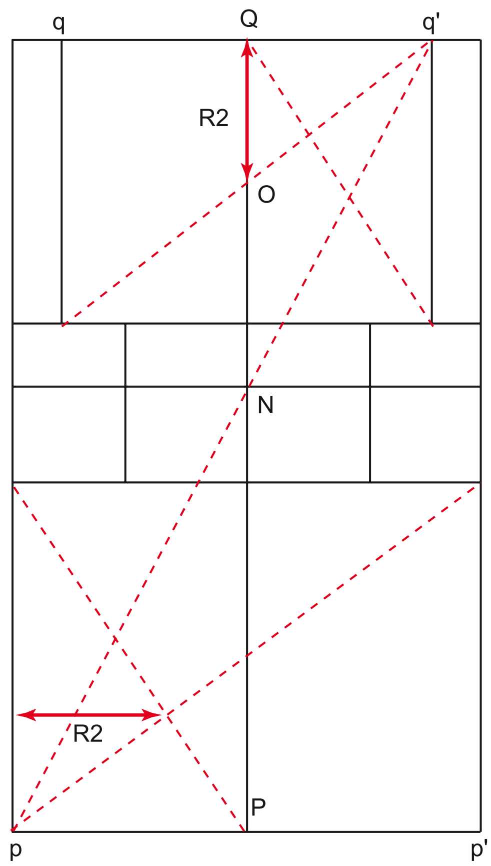
Outline of viola N6 superimposed on its geometrical reconstruction (in red).

The lower part is drawn by four arcs with the following radii:
R1 *= NP* ***R2*** *= 1/2 of XP* ***R3*** *= XP* ***R4*** *= 1/3 of XN*

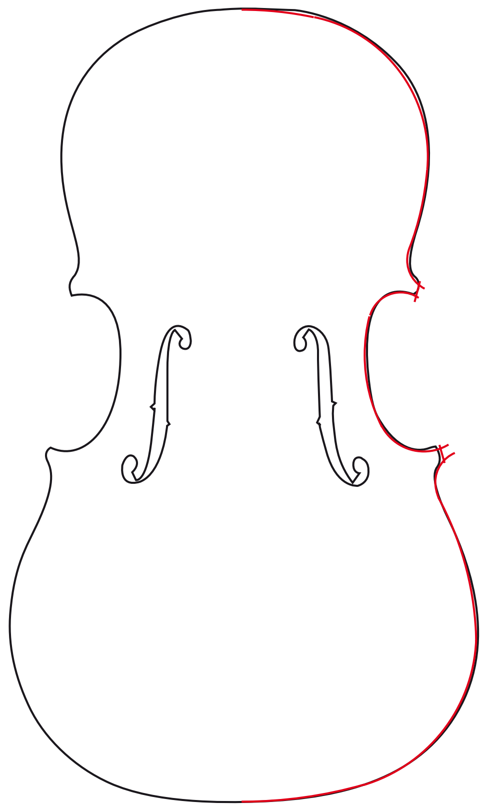
The midpart is drawn by three arcs with the following radii:
R1 *= XZ* ***R2*** *= 1/2 of XN* ***R3*** *= 1/2 of NZ*

The upper part is drawn by four arcs with the following radii:
R1 *= NQ* ***R2*** *= 1/2 of ZQ* ***R3*** *= ZQ* ***R4*** *= 1/3 of XN*

Viola N18



In this last example, the radii R2 measure one-third of pp' below and half of ZQ above.



Outline of viola N18 superimposed on its geometrical reconstruction (in red).

The framework having been defined, the lower part is drawn by four arcs with the following radii:
R1 *= pp'* ***R2*** *= 1/3 of pp'* ***R3*** *= 2/3 of pp'* ***R4*** *= 1/2 of NZ*

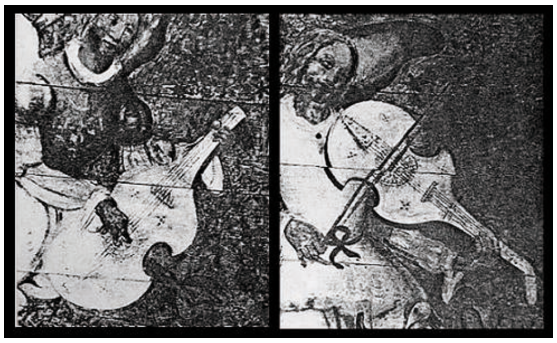
The midpart is drawn by three arcs with the following radii:
R1 *= XZ* ***R2*** *= 1/2 of XN* ***R3*** *= 1/2 of NZ*

The upper part is drawn by four arcs with the following radii:
R1 *= NQ* ***R2*** *= 1/2 of ZQ* ***R3*** *= ZQ* ***R4*** *= 1/2 of NZ*

Notes on the making process

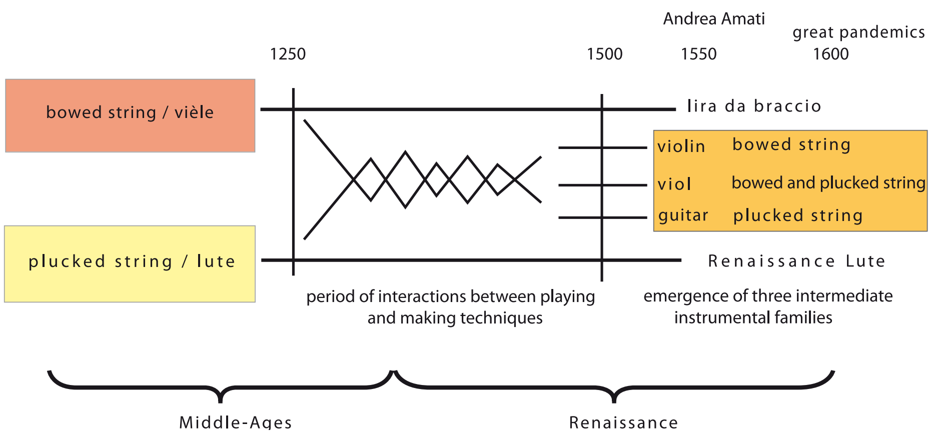
It was in 13th century Spain, in the cultural melting-pot of AlAndalus, that bowed string instruments, omnipresent in the Christian world, started to mix with the plucked string instruments emblematic of the Islamic tradition. From then on, musicians began to use both techniques on the same instruments. This practice was at the origin of the cutaways that appeared on instruments inspired by Islamic models and henceforth called vihuela *del arco* or *da mano*. At the same time, the simple design of medieval bowed instruments (three elements, carved and assembled) came into contact with the more elaborate technique used by lute-makers. Lute soundboxes were not carved from a block of wood but assembled from skilfully adjusted strips. In the 15th century, the two methods of construction corresponding to two styles of playing (plucked and bowed) began to mingle. These combinations gave rise in the following century to the intermediate families of viols, violins and guitars. The pictorial record bears witness to this complex period of transition, showing instruments suited to both plucking and bowing.[13]

The relic of Saint Catherine is one of the oldest string instruments to have survived intact. A unique record of that transitional period in which the intermediate families of instruments were made, it combines both the form and the playing style of two types of instrument. Frederico Zuccari's 1608 painting seems to show the relic with two bridges, and a dual playing technique with the right hand on the bow and the left hand in the position of a spread chord.



Example of an instrument that can be played both ways (15th century, France).

This interaction of playing styles and cultures also gave rise to the different types of making that existed in the early 16th century. At that time in Europe, bended ribs had entirely replaced the Gothic "saw-out" technique, though in some places (like Brescia) for a while the overall design remained that of the mediæval bowed instrument. In contrast, the techniques used by lute-makers penetrated more deeply in Cremona, where the use of a mould and a neck nailed to a soundbox was transposed to violin-making.



Interaction of cultural and technical factors giving rise to new families of instruments.

Conclusion

This study shows that Brescian instruments perpetuated principles already in use in the Renaissance. Surfaces are correctly constructed using simple measurements that incorporate geometric constructions based on diagonals. As the tables in the annex show, the match between the actual dimensions of the instruments and their theoretical values is surprisingly close. This finding indicates that the coherence of Brescian instruments was at least equal to that of the Amati's' output and greater than that of a good deal of later Cremonese work. The marked differences between the instruments produced in the two cities are therefore not attributable to the quality of the craftsmen's knowledge but to the ways in which they put it to use.

The measurements apply to the definition of an exterior outline, not to the construction of an interior form. This point supports the argument that instruments were made without a mould, a technique that owed more to the Gothic model of bowed instruments than was the case for Cremonese instruments of the same period. In accordance with the Ancients' particular conception of symmetry, the Brescian instrument-makers paid more attention to coherence of measurement than to the outline as such.

A study of these models reveals a treasure-trove of geometric ingenuity and a degree of elaboration that is in contradiction with the more archaic aspects of Brescian production. It suggests that the violin did not suddenly appear from nowhere but was the result of three centuries of evolution in instrument-making, spanning the whole of Europe and its history.

This distant past, gradually revealed, gives more than the measure of a rich and little-known heritage. It encourages instrument-makers to see the future of the violin in the continuation of the knowledge that gave the instrument its worth and not only in sciences alien to its nature.

Notes

1 For a comprehensive analysis of Henri Arnaud de Zwolle's drawing, see François Denis, *Traité de lutherie*, ALADFI, Nice, 2006, pp 48-56.

2 If a width qq' =158 mm is compared with another width pp' =197.5 mm, the relation between the two widths is expressed by the ratios 158/197.5 = 0.8 or 200/160 = 1.2. These quotients, 0.8 and 1.2, mean that the relation between the two magnitudes is like that of the numbers 4 and 5.

3 Ibid. note 1.

4 On the empirical use of Amati family forms by Stradivarius and Guarneri del Gesu, see Analyse de la forme ....(to be publish)

5 On this approach to the plan, it is instructive to read Vitruvius, On Architecture, Book III, "Definition and necessity of symmetria", and Albrecht D¸rer, Géométrie, translated and introduced by Jeanne Peiffer, Seuil, Paris, 1995, p 139.

6 These ratios are also what we call comparisons. At the time, there was no conception of a ratio without a physical link between the comparable objects. It follows that a "remote" comparison has no real meaning, and for that reason relative measurements always concern adjacent dimensions. Although this principle no longer has any prevalence, it is nevertheless crucial to the conception of form in previous eras.

7 The relations discussed here have been taken from the measurements given in the attached tables and provided by Mr E. Blot and Mr Y. Gateau. The instruments considered in this study are those whose original dimensions have not been altered.

8 In this case a rectangular surface that can be constructed geometrically by the half-diagonal of the square or one of its approximate measurements (e.g. 4/7 = 0.571 or 7/12 = 0.583 (the most common approximations are given in Table 1 in the annex). Manipulation of these values played a decisive role in the origin of the measurements. On these fundamental notions for the history of measurement, see François Denis, op. cit., pp 25-56. Louis Frey, "Données architecturales et hypothèse sur la mathématique pré-euclidienne", in *Bulletin Antike Beschaving* (BABesch), 1989, pp 90-99.
Pierre Gros, "La géométrie platonicienne de la notice vitruvienne sur l homme parfait" in *Rivista del Centro internazionale di Studi di Architectura Andrea Palladio di Vicenza*.

9 About the correlation between Fibonacci series and measurements see François Denis, *Traité de lutherie*, ALADFI, Nice, 2006, pp 21-56.

10 Dividing a space between two vertical lines a whole number of times is easy if you have a graduated ruler. For an illustration of the method, see François Denis, op. cit., p 52.

11 This is the extension of the Quadrivium sciences as presented by Proclus. The geometry of movement par excellence is that of astronomy and of the trajectory of the planets, while "static" geometry applies to surfaces and figures. These two sciences are completed by arithmetic, which deals with the manipulation of whole numbers, and music, which expresses the rightness of the relations between whole numbers in audible form.

12 On the compass techniques used, see François Denis, op. cit., pp 101-113.

13 Christian Rault, *Géométries médiévales, tracés d'instruments et proportions harmoniques*, in *Instruments à cordes du Moyen Age*, Proceedings of the Royaumont conference, 1994, ed. Créaphis, Grânes (France), pp 51-56; *Los instrumentos musicales en el siglo XVI* in *Encuentro Tomás Luis de Victoria y la musica española del siglo XVI*, Avila, UNED, May 1993, pp 231-242; *Les modifications structurelles radicales des instruments à cordes au XVI° siècle*, in Pastel no. 21, September 1994, pp 30-36.

Annex

Table 1

Most frequent ratios with the name of their proportions

3/8 = 0,375	geometric section
2/5 = 0,400	harmonic or geometric section
√2-1 = 0,414	harmonic section
3/7 = 0,429	harmonic section
4/9 = 0,444	harmonic or geometric section
4/7 = 0,571	harmonic section
5/9 = 0,555	harmonic or geometric section
2-√2 = 0,586	harmonic section
3/5 = 0,600	harmonic or geometric section
5/8 = 0,625	geometric section
2/3 = 0,666	arithmetic section

Table 2

	N2V	N14V	Theoritical ratios	N3V	N11V	N12V	Theoritical ratios	N13V
PX to PQ	0,413	0,413	√2-1=0,414	0,415	0,414	0,429	3/7 = 0,429	0,422
ZX to XQ	0,402	0,396	2/5 = 0,400	0,394	0,406	0,375	3/5 =0,375	0,395
pp' to PQ	0,568	0,585	2-√2 = 0,586	0,577	0,589	0,570	4/7 = 0,571	0,586
pp' to XQ	0,967	0,997	1,000	0,986	1,006	0,997	1,000	1,014

Table 3

	N2V	N14V	Theoritical ratios	N3V	N11V	N12V	Theoritical ratios	N13V
aa' to pp' (top)	0,640	0,620	5/8 = 0,625	0,624	0,604	0,602	3/5 = 0,600	0,624
bb' to pp' (top)	0,222	0,187	3/16 = 0,187	0,233	0,195	0,200	1/5 =0,2	0,199
pp' to PQ	0,568	0,585	2-√2 = 0,586	0,577	0,589	0,570	4/7 = 0,571	0,586
pp' to XQ	0,967	0,997	1,000	0,986	1,006	0,997	1,000	1,014

Table 4

	N2V	N14V	Theoritical ratios	N3V	N11V	N12V	Theoritical ratios	N13V
wf to hf	0,516	0,502	1/2 = 0,5	0,518	0,559	0,501	1/2 = 0,5	0,589
hf to aa'	0,633	0,694	7/10 = 0,700	0,604	0,606	0,666	2/3 = 0,666	0,579

Table 5

	N5A	Theoritical ratios	N6A	Theoritical ratios
PX to PQ	0,414	√2-1 = 0,414	0,410	√2-1 = 0,414
ZX to XQ	0,385	3/8 = 0,375	0,396	2/5 = 0,400
pp' to PQ	0,568	4/7 = 0,571	0,573	4/7 = 0,571

Table 6

	N5A	Theoritical ratios	N6A	Theoritical ratios
hf * to aa'	0,668	2/3 = 0,666	0,657	2/3 = 0,666
bb' to aa'	0,374	3/8 = 0,375	0,387	3/8 = 0,375
aa' to pp' (table)	0,603	3/5 = 0,600	0,636	5/8 = 0,625

(* longitudinal lenght of the f-holes)

Table 7

	N17A	Theoritical ratios	N18A	N10 cello	Theoritical ratios
PX to PQ	0,443	4/9 = 0,444	0,439	0,447	4/9 = 0,444
ZX to XQ	0,354	5/14=0,357	0,358	0,361	5/14=0,357
pp' to PQ	0,583	2-√2 = 0,586	0,591	0,602	3/5=0,600

Table 8

	N17A	N18A	N10 cello	Theoritical ratios
ZX to XQ	0,354	0,358	0,361	5/14=0,357
ZP to PQ	0,360	0,360	0,353	5/14=0,357

Table 9

	N17A	N18A	Theoritical ratios
aa' to pp' (top)	0,558	0,533	5/9 = 0,555
bb' to pp' (top)	0,232	0,219	2/9 = 0,222
ee' to qq'	0,665	0,663	2/3 = 0,666