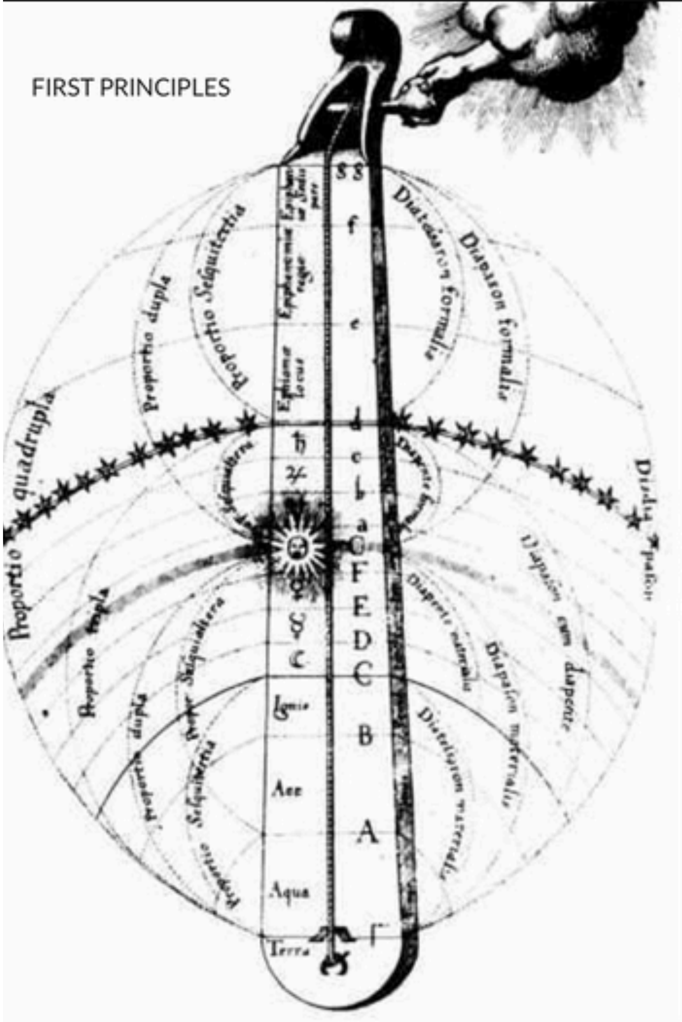


Left The hand of God comes down to tune the 'lyre cosmique', a monochord encompassing the music of the spheres, in this illustration from Robert Fludd's 1617 encyclopaedia *Utriusque cosmi, maioris scilicet et minoris, metaphysica, physica atque technica historia*

MUSIC OF THE SPHERES

In an age of little numeracy or literacy, how did luthiers settle on the proportions of stringed instruments, with hardly any variation in their basic design? **François Denis** shows how the principles of the classical Greeks – notably Pythagoras – informed their thinking

According to tradition, the first person to identify the correlation between musical harmony and whole numbers was the Greek philosopher Pythagoras. It is said that he was passing a blacksmith's forge one day, and noticed how all the smith's hammers made different sounds when striking the anvil. Using this as a starting point, he came up with the idea that sound could be connected to whole numbers. Furthermore, it was also the Greeks who determined the link between arithmetic and ideas of beauty, grounding the notion that numbers can be used to explain the laws of nature, on both a micro and a macro level.

On the face of it, Pythagorean philosophy may not seem to have any connection with violin making, but its principles shed light on various approaches to measurement that are commonly believed to have originated from the makers' trial and error. This article will show how Pythagorean systems of measurement, connecting lengths with musical intervals, informed the design of early stringed instruments. It also attests to the importance of this way of thinking about measurement, a way that has for so long been buried in the past.

A method of using the monochord, as depicted in Lodovico Fogliano's 1529 treatise *Musica theoretica*

ALL IMAGES COURTESY FRANÇOIS DENIS

The emblematic tool for explaining the musical world through numbers is an archaic chordophone called a 'monochord'. It is made from a simple string stretched over a resonator. It can be divided into two parts which, made to vibrate, will produce two sounds. The monochord is used to prove Pythagoras's claims that harmony is a mix of two quantities, depending on whole-number relationships – essentially that our capacity to discern the quality of chords and musical relations is governed by whole numbers.

When we bisect the string of the monochord into two equal parts (**figure 1**), we hear the same note on both sides of the bridge: a unison (i.e. the ratio is 1:1). These notes and the open string are an octave apart (1:2).

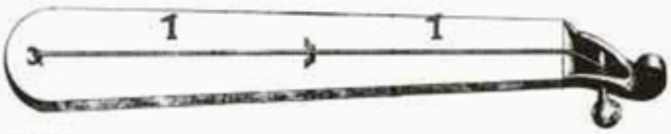


FIGURE 1

Whenever a string is divided into two different parts it actually generates *three* intervals. If we call the string's full length L, the moving bridge will split this distance into two parts, respectively called S (short length) and M (medium length). In each case, L = S + M (**figure 2**). Thus, the relationships S to L, M to L and S to M are all created by placing the bridge at a given position.

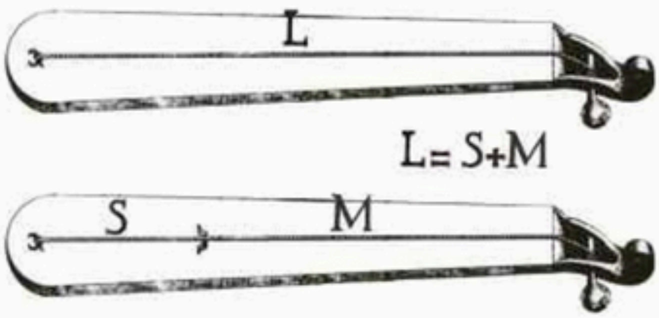


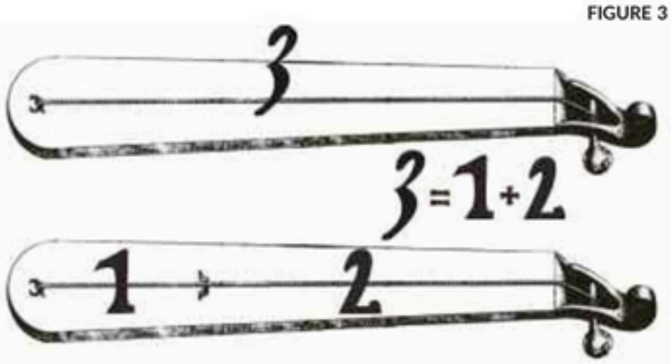
FIGURE 2

Figure 3 shows how the above values can be equally distributed, i.e. that L = S + M such that 3 = 1 + 2. This generates the following pitch relationships:

- S to L – 1 to 3 = a 5th above the octave
- M to L – 2 to 3 = a 5th
- S to M – 1 to 2 = an octave

Anything you want to build needs measurements. In the past, these measurements (in the sense of 'a series of dimensions') were not established as we are used to them now. Instead, architects used a principle called 'extraction', wherein each measurement is the result of dividing one that preceded it. Equally, like music, each new measurement then becomes a reference for another one. Knowledge of this extraction process actually conditions the viewer's perception of the finished product – there is an 'order' to know, if you want to have a deep understanding of what you see.

FIGURE 3

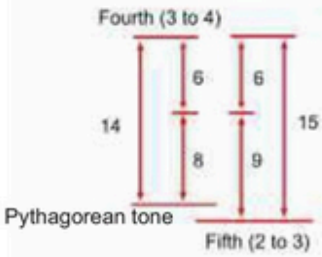


In its primary sense, 'harmony' describes an intimate relationship wherein two parts merge pleasingly into a whole. In early music, harmony is the consonance of unison (1 to 1), octave (1 to 2), 5th (2 to 3) and 4th (3 to 4) relationships. Hearing these consonances on a monochord requires dividing the string into 2, 3, 5 and 7 equal parts.

The interval of the so-called 'Pythagorean' tone is the distance that separates the 5th from the 4th. As explained above, the numerical expression of the pitch interval then depends on the part taken as a reference of the calculation in the equation L = S + M.

Figure 4 shows the relationship between the 4th (ratio 3:4) and the 5th (ratio 2:3). To illustrate the relationship most clearly, we use a value for S (short length) that is divisible by both 2 and 3. The simplest value is 6 (which is 2x3), so we expand 3:4 to 6:8, and 2:3 to 6:9. This gives L a value of 14 for the 4th and 15 for the 5th.

FIGURE 4



If these same ratios are expressed from M in the equation L = S + M, M must be divisible by both 3 and 4, the simplest value being 3x4 which is 12. So for the 4th we have 20 (= 8+12) and for the 5th we have 21 (= 9 + 12) (**figure 5**).

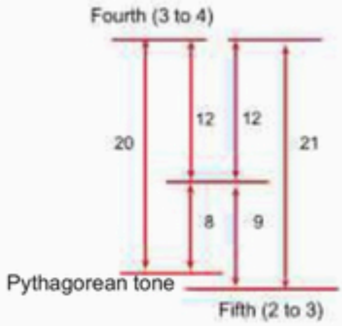


FIGURE 5

Finally, if L is the reference length of the calculation in the equality L = S + M: L must be divisible by 5 (which is 2+3), ▹

Right The Italian composer Gaffurius explains music and arithmetic to students, in this illustration from his 1518 book *De harmonia musicorum instrumentorum*. It shows that 4 is the harmonic proportional mean of 3 and 6, having a relationship of a 5th with 6, a 4th with 3, and that 3 has a relationship of an octave with 6.

INSTRUMENT MAKING WAS LIKE MUSIC: EACH NEW MEASUREMENT BECAME A REFERENCE FOR ANOTHER ONE

and 7 (3+4), so we have L = 35 (5x7). **Figure 6** shows these proportions: 35 = 14+21 and 35 = 15+20.

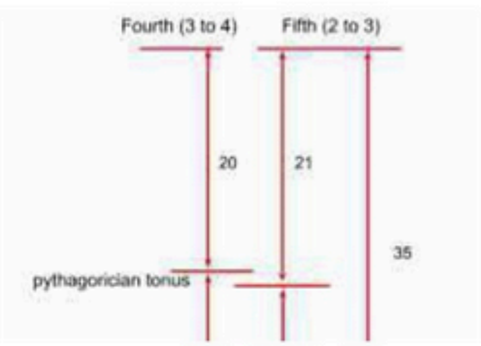


FIGURE 6

So, these three expressions of the difference between the 5th and the 4th can be represented in a single scale of measurement (**figure 7**) where the interval of the tone is a unit of measurement. It can be seen from this diagram that the smallest interval, which we call the 'Pythagorean tone', has a value of 1/35.

It turns out that these fractional tone values, derived from the difference between the 5th and the 4th, play an important role in the variations of the instruments in the violin family – as we shall now see.

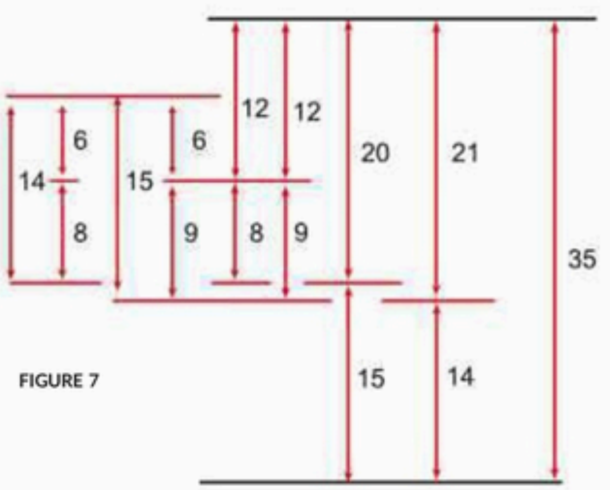
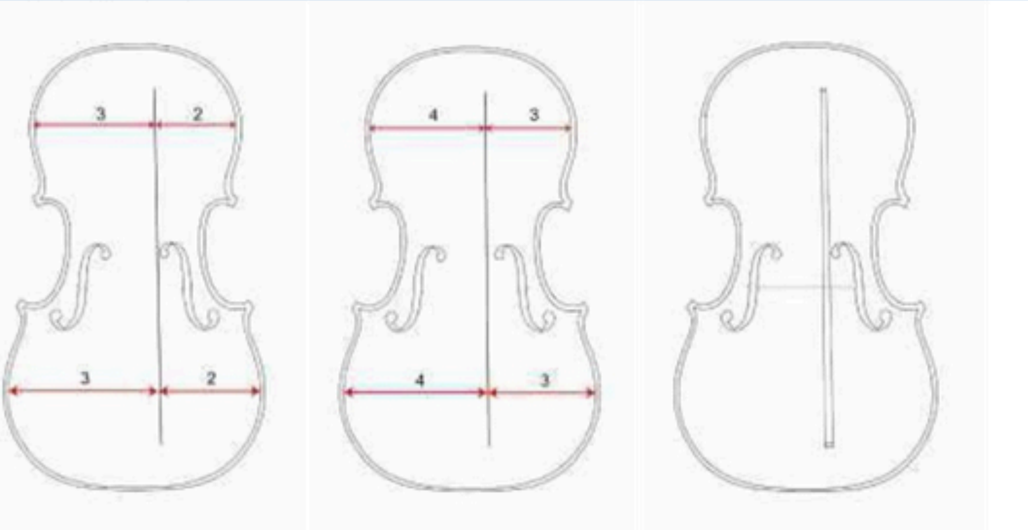


FIGURE 7

THE INTERVAL OF TONE EXPLAINS HOW THE DIMENSIONS OF THE VIOLIN FAMILY CAN VARY

THE POSITION OF THE BASS-BAR

Luthiers are used to positioning the bass-bar so that it is 4/7 of the width – four parts from the bass side and three from the treble side (**figure 8**). If we use a ratio of 3 to 2 instead of 4 to 3, applying a division of the 5th (**figure 9**), we can determine a pitch interval ▸



FIGURES 8–10 show that the standard width of the bass-bar is 1/35 the standard width of the violin

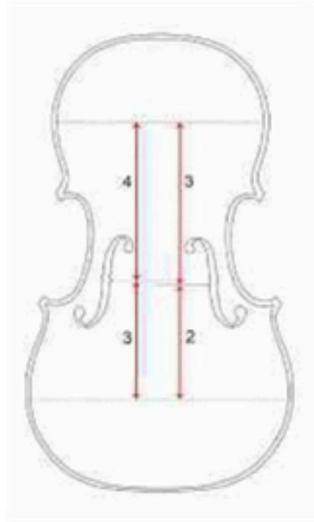


FIGURE 11

of 1/35 (**figure 10**). For the standard violin measurements (160mm for the upper bouts and 200mm at the lower bouts) we find that the Pythagorean tone interval corresponds to 5.5mm at the bridge position – which is the standard average width of a violin bass-bar.

To summarise: the bass-bar bisects the top plate by spanning the interval that separates two harmonic consonances. Perhaps this could explain its name in French: *la barre d'harmonie*.

> THE PLACEMENT OF THE BRIDGE RESULTS FROM A BISECTION OF THE LENGTH OF THE INSTRUMENT, OR A PART OF THAT LENGTH

THE PLACEMENT OF THE BRIDGE

The setting of the bridge to align with the lower notches of the f-holes became standard during the 17th century. However, depictions of instruments before that time show that other placements have also been used. The subject goes beyond the limited scope of this article, but the basic principle is that the placement of the bridge results from a bisection of the length of the instrument or a part of that length. In the violin family this reference length (L) corresponds to the difference between the largest widths of the upper and lower bouts (**figure 11**). The variation of the placement found in each family also corresponds to the interval of the Pythagorean tone.

THE POSITION OF THE CORNERS

Another example of variation based on this 1/35 measurement is in the positioning of a stringed instrument's corners, which is frequently dictated by the placement of 4ths, 5ths and 7ths. **Figures 12–14** give examples of this in three instruments of differing sizes. It is interesting to note that the proportions of the Brothers Amati

FIGURE 12 Antonio Stradivari 'Davidov' cello 1712

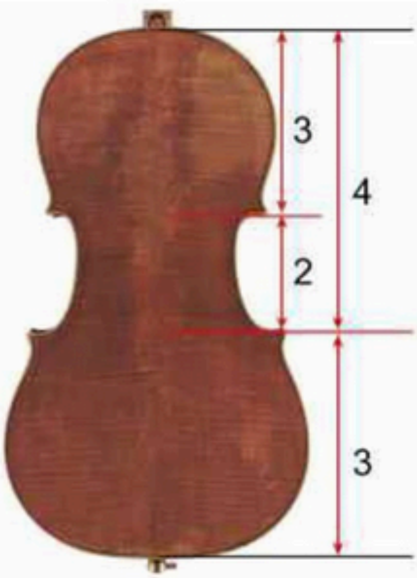


FIGURE 13 A 1580 Gasparo da Salò viola

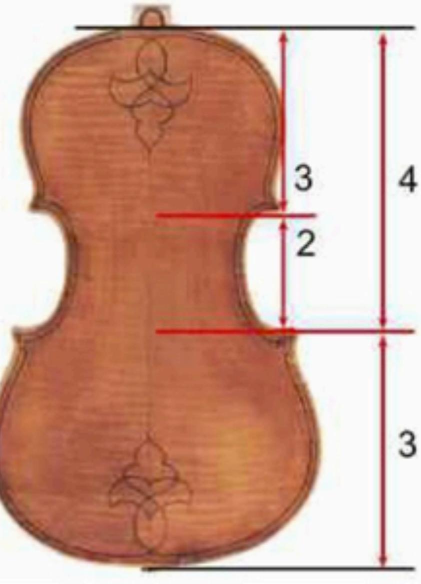
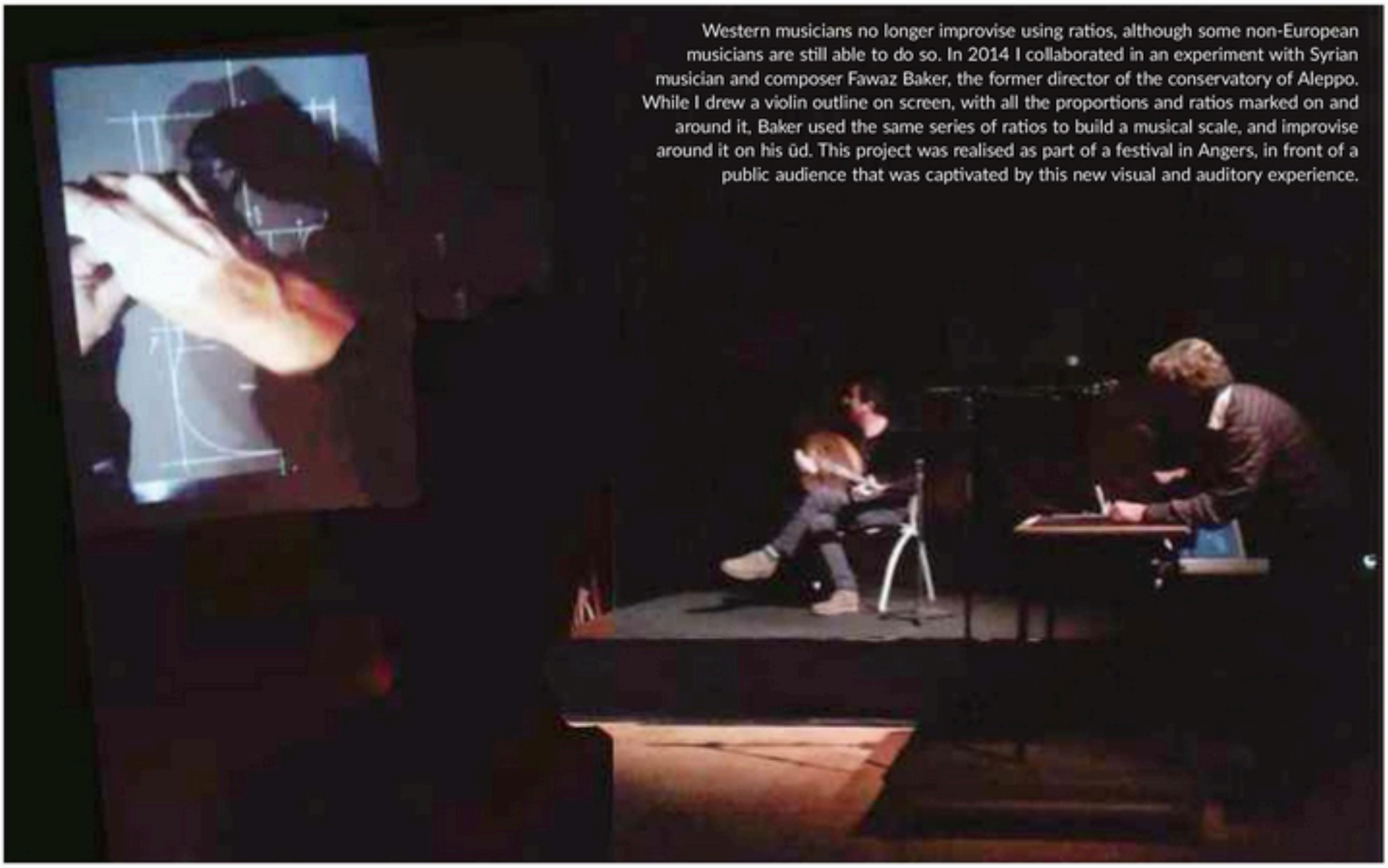


FIGURE 14 A Brothers Amati tenor viola of 1592



Western musicians no longer improvise using ratios, although some non-European musicians are still able to do so. In 2014 I collaborated in an experiment with Syrian musician and composer Fawaz Baker, the former director of the conservatory of Aleppo. While I drew a violin outline on screen, with all the proportions and ratios marked on and around it, Baker used the same series of ratios to build a musical scale, and improvise around it on his ūd. This project was realised as part of a festival in Angers, in front of a public audience that was captivated by this new visual and auditory experience.

USING THESE PROPORTIONS AS STANDARD MEASUREMENTS COULD HAVE BEEN A USEFUL AIDE-MEMOIRE FOR LUTHIERS AT A TIME WHEN WRITING WAS HARDLY USED

Andrea Amati made two sizes of violins whose difference is 1/35 of the length

tenor viola (3 to 2 and 4 to 3) are the reverse of those of the 'Davidov' Stradivari cello and Gasparo da Salò viola (4 to 3 and 3 to 2).

Using these proportions as standard measurements could have been a useful aide-memoire for luthiers at a time when writing was hardly used, and contributed to the recognition and understanding of the various patterns and moulds.

CONCLUSION

Not all variations in measurement can be related to the proportions used in ancient Greek studies. There are other ratios that occur frequently – 5 to 7, 5 to 8, 7 to 4 and so on. In my 2006 book *Traité de Lutherie* I demonstrate how these ratios are all founded on an elaborate system of proportions, using Arnault de Zwolle's famous 15th-century diagram of a lute as the basic model. What is clear from this is that our appreciation for the works of classical antiquity is founded on the principles of harmony and proportion that result from close observation and experience of the natural world. This experience preceded the discovery of mathematical theorems, and formed the basis of artistic endeavour up to the Early Modern purpose era. As the great architect Andrea Palladio said in his 1570 treatise *I quattro libri dell'architettura*: 'As the proportions of voices are harmony for the ears, so the proportions of measurements are harmony for the eyes. Such harmonies are often very pleasing without anyone knowing why – with the exception of those investigating the causes of things.' In this article, we have tried to share the experience advocated by that great man. ●