PYTHAGOREAN TONUS AND LUTHERIE

Abstract:
Measurements of distance have long been related to sense perception and to harmonic musical intervals. This article draws attention to an important sense and understanding of measurement that is buried in the past.

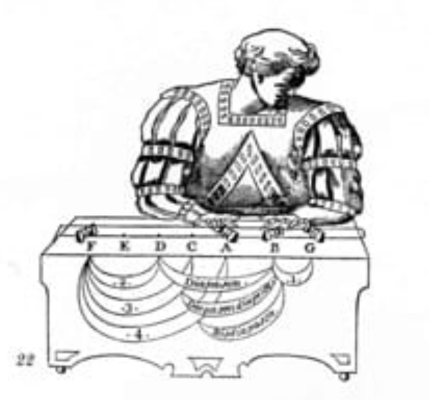


fig 1

Method to use the monotone of "Musica teorica" treated of Lodovico Fogliano Venice, 1529.

INTRODUCTION

Some sounds blend better than others and our ears are the guide. According to tradition, Pythagoras is the first to establish a link between harmony and whole numbers. The association of arithmetic and ideas of beauty grounds the notion that numbers are embedded in nature and explain its laws: from microcosm to macrocosm.

Pythagorean philosophy may not seem to have any connection with violin making, but its principles do shed light on various approaches to measurement that are commonly believed to originate only in practice.

Following the Pythagorean sense of measure that connects lengths with musical intervals, the shapes of baroque instruments are more than outlines, they reveal musical themes as series of modulations made of notes.

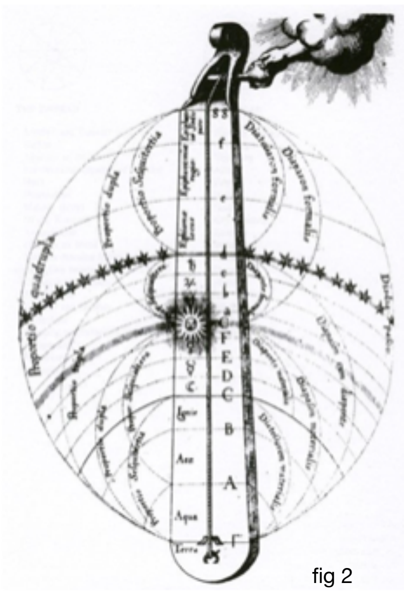


fig 2

Robert Fludd, Utriusque cosmi majoris scilicet et minoris metaphysica, physica atque technica historia, Oppenheim, J. T. De Bry, H. Galleri, 1617-1624, I, 1, Livre III, Chap. 3, p. 90.

THE MONOCHORD

The emblematic tool for explaining the world through numbers is an archaic chordophone called the"monochord". It i made of a simple string stretched over a resonator.

It can be divided into two parts by means of a moving bridge and when plucked or bowed theses two parts produce two sounds. Its use proves some of Pythagoras’s claims about integers and music. Our capacity to discern the quality of chords and musical relations is actually governed by whole numbers.

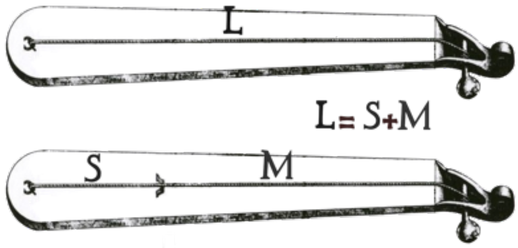
IDEAS ABOUT THE BISECTION OF A STRING

When we bisect the string of the monochord in two equal parts, we hear the same note on both sides of the bridge: a unison. (ratio 1-1 fig 3). These notes and the open string are an octave. (1-2).

fig 3

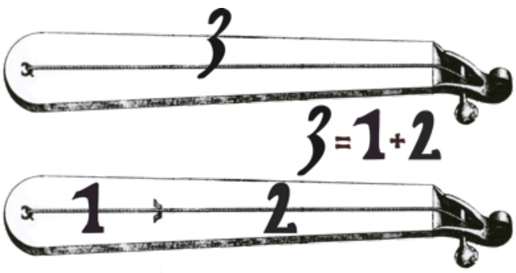
Whenever a string is divided into two parts it actually generates three intervals. If we call L the longest length of the chord, the moving bridge will divide this distance into two parts respectively called S and M. In each case L = S + M (fig 4). Thus the relations S to L, M to L and S to M are all created by simply placing the bridge at a given position.

fig 4



If we follow the example (fig 5) of a division of the string into 3 equal parts we have L = S + M such that 3 = 1 + 2, which generates the following harmonic relations:
S to L : 1 to 3 is a fifth above the octave,
M to L : 2 to 3 is a fifth and
S to M : 1 to 2 is an octave.

fig 5



A diversity of intervals, ratios and magnitudes here depend on a single act, bisecting a string with a bridge. This reminds us that all measurement implicitly refers to the process that established it. Architectural manuals from the Middle Ages do not fail to insist on this point. For them, measurement comes from a process of "extraction" that must be understood in order to grasp the constructive logic of the visual world. Proportionality and measurement thus refer to the movement of the eyes as well as the rational movement of thought.

In many languages a polysemy - the coexistence of many possible meanings for a word or phrase- exists between hearing and understanding. In English "sound" can refer simply to something heard, but also to the perception of an intended meaning. (The "sound" of the the violin. That "sounds" correct.) In French "entendre" comes from Latin "intendere" meaning to direct the eyes, attention or mind. In this way "hearing" in French, also means understanding what we see. This elucidates a connection between modulations of sound and the movement of the eyes. But the march of progress from the seventeenth century onwards drew academics away from these fundamental relations. From that point on, a full experience of the sensible coherence of works from antiquity is less and less available. The notorious "quarrel between the ancients and the moderns" thus begins.

Before the progress of science was tied to industry –a defining element of modernity- what was seen related directly to what was understood. There is an acute, lost sense we are dealing with here and by "sense", we must understand perception, meaning and order. This broad, inclusive conception of perception and reason is repeated by Alberti when he states that "numbers that create captivating sounds in our ears are the same ones that please our eyes and our minds" (Wittkower p133 Alberti).

fig 8

Illustration of the teaching of music and arithmetic. It shows up that 4 is the harmonic proportional mean of 3 and 6 having a relation of fifth with 6, a relation of forth with 3 and, that 3 has a relation of octave with 6. *(Gaffurius and his pupils. From harmonia musicorum instrumentorum opus 1518)*

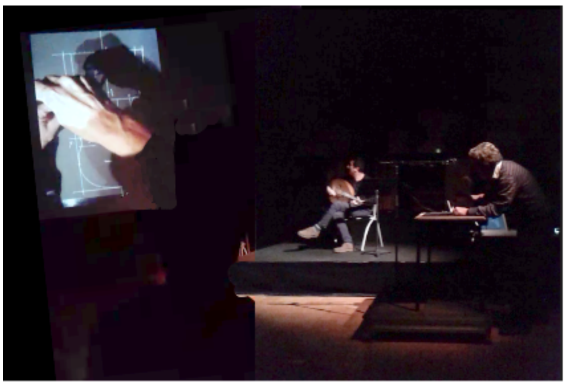


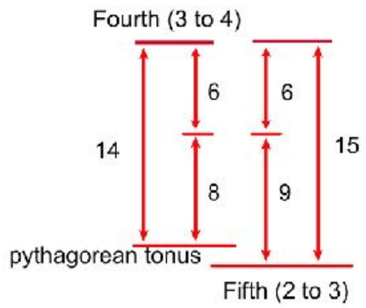
fig 9

Western musicians no longer improvise using ratios although some non-European musicians are able to do so. Fawaz Becker, former director of the conservatory of Aleppo is one of them. He agreed to participate in the following experiment. A violin was drawn live by indicating the ratio used for the construction. Simultaneously, the musician used the same series of ratio to built a musical scale and play music with them on his ûd. This project was realised in front of a public captivated by a new visual and auditive experience.

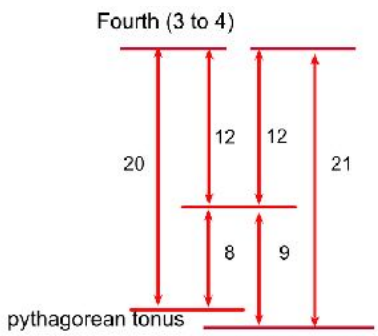
THE PYTHAGORIAN TONE IN LUTHERIE

We have seen that hearing these consonances on a monochord requires dividing the unique string into equal parts ; 2 (for unisson), 3 (for octave), 5 (for the fifth) and 7 (for the fourth)
The interval of the so-called "Pythagorean" tone is then the distance that separates the fifth from the fourth. As explained above, the numerical expression of the pitch interval then depends on the part taken as a reference of the calculation in the equality L = S + M.

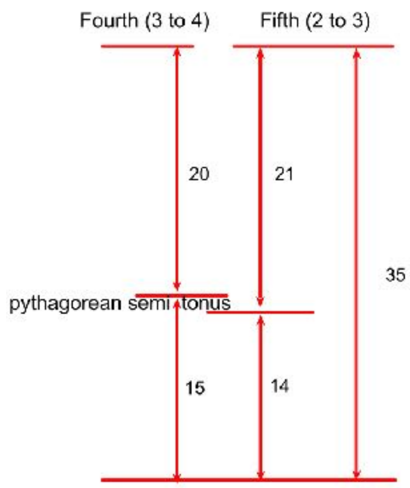
If the ratios of forth (3 to 4) and fifth (2 to 3) are expressed from S. In the equality L = S + M: S must be divisible by 2 and 3 so we have S = 2x3 = 6 such that 14 = 6 + 8 and 15 = 6 + 9 where the ratio of 8 to 9 is the so-called pythagorean tonus.

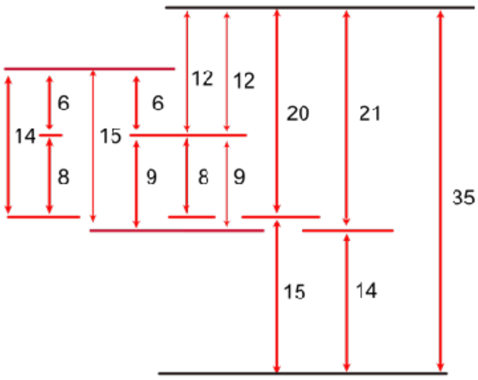


If these same ratios are expressed from M. In the equality L = S + M, M must be divisible by 3 and 4 so we have M = 3x4 = 12 such that 20 = 8 + 12 and 21 = 9 + 12. We find again the ratio 8 to 9 of the tonus set between the fifth and the fourth.



Finally, if L is the reference length of the calculation in the equality L = S + M: L must be divisible by 2 + 3 = 5 and 3 + 4 = 7 so we have L = 5x7 = 35 such that 35 = 14+21 and 35 = 15 + 20.
Ratios 14 to 15 and 20 to 21 are there two values of the semi-tonus





These three expressions of the difference between the fifth and the fourth can be represented in a single scale of measurement where the interval of the tone plays the role of a unit of measurement

It turns out that these fractional tone values, derived from the difference between the fifth and the fourth, play an important role in the variations of the quartet's instruments. What we will demonstrate

SI VELIS PROBARE…

THE POSITION OF THE BASSBAR

We are used to placing the bassbar by a division of the fourth: (4-3) widths of the top and the bottom bouts (which equals 1/7 of the half width) (fig 11). If a division of the fifth (3-2) is applied in the same places (FIG. 12), a pitch interval (1/35) is determined (FIG. 13). For standard measurements 160 mm at the top and 200 mm at the bottom we find that the Pythagorean tone interval between the fourth and the fifth equals the 5.5 mm at the bridge position, which is the standard average measurement of a violin bassbar.
To summarize, the bassbar bisects the table by spanning the interval that separates two consonances of harmony. This could perhaps explain its name in French "barre d'harmonie".

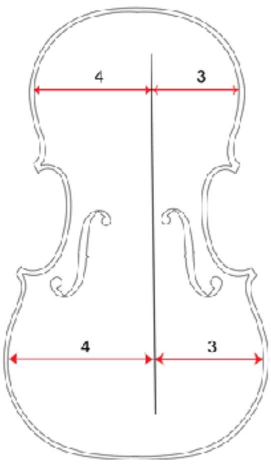


fig 11

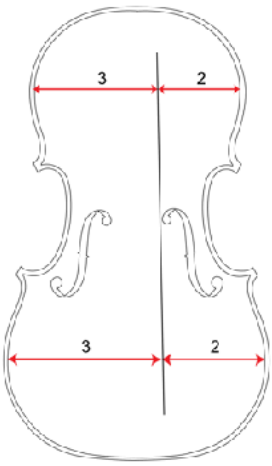


fig 12

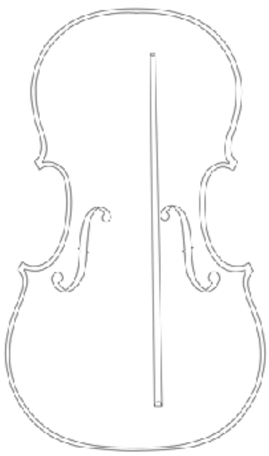


fig 13

THE PLACEMENT OF THE BRIDGE

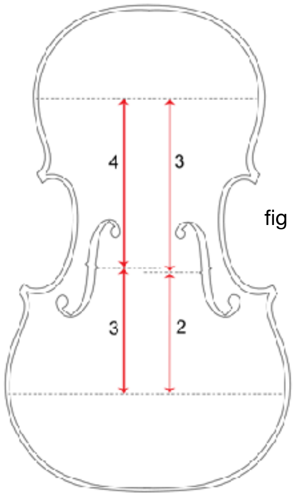


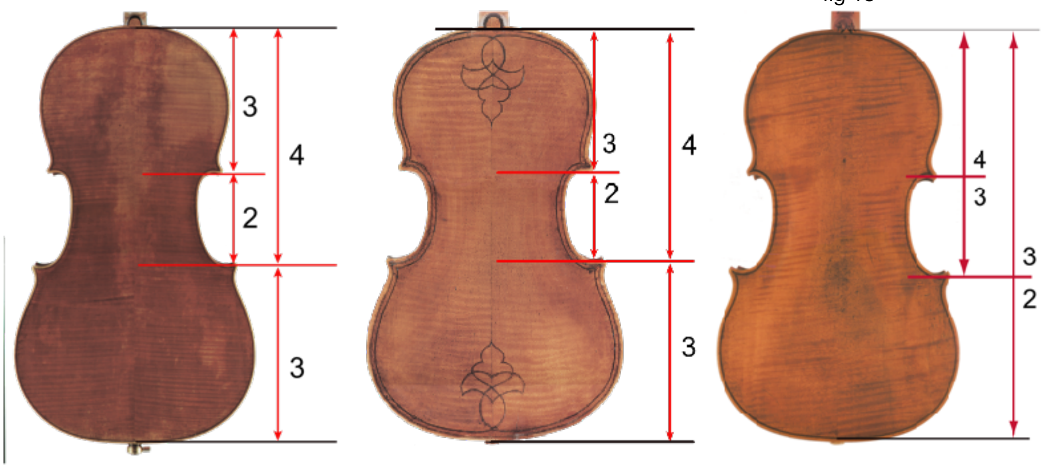
fig 14

The setting of the bridges at the notches of the "f" became a standard during the seventeenth century. Iconography shows that other placements have been used. The subject goes beyond the limited scope of this article, but the basic principle is that the place of the bridge results from a bisection of the length of the instrument or a part of that length. In the violin family this reference length (L) corresponds to the difference between the largest widths of the top and the bottom. The variation of the placement found in each family also corresponds to the interval of the Pythagorean tone.

THE POSITION OF THE CORNERS

The position of the corners of stringed instruments is another example of variations based on a tone interval. Indeed, the corners are frequently placed by fifths and fourths.

fig 16 fig 17 fig 18



Example of partitions according to the consonances of the harmony. The Davidoff (cello of Stradivarius) (fig 16) is organized in the length on the series fourth and fifth while the alto of Gasparo da Salo (fig17) and the tenor of the Amati brothers (fig 18) reverse this relation. These systems facilitate the recognition and understanding of forms and for these reasons they have been a useful mnemonic aid in a craft society with little or no use of writing

MODULE AND TONE

The presence of a unit value related to the concept of the module is another consequence of the use of the harmonic divisions. For the record, a module is a determined quantity capable of setting proportional relations. In the case described above the tone is indeed a fixed quantity common to relations. A simple calculation shows that the fraction 1/35 of the total length is a modular value in the sense that this fixed interval is found an integral number of times in each dimension and in their proportionate variations.

We note that the ratio of tone of 8 to 9 of the "C" bout allows the passage of the fourth (12 to 8) to the fifth (12 to 9) (fig 19). Etymology reinforces this idea. Module comes from the Latin modulus, modus meaning cadence and measure while its Greek equivalent is the word τόνος which means precisely the interval of the tone. This meaning recurs in the word modulation which in music means a change of tone. Although proportional modulations found in musical instruments are much more limited than modulations in music, the analogy between the two is evident.

.

fig 19

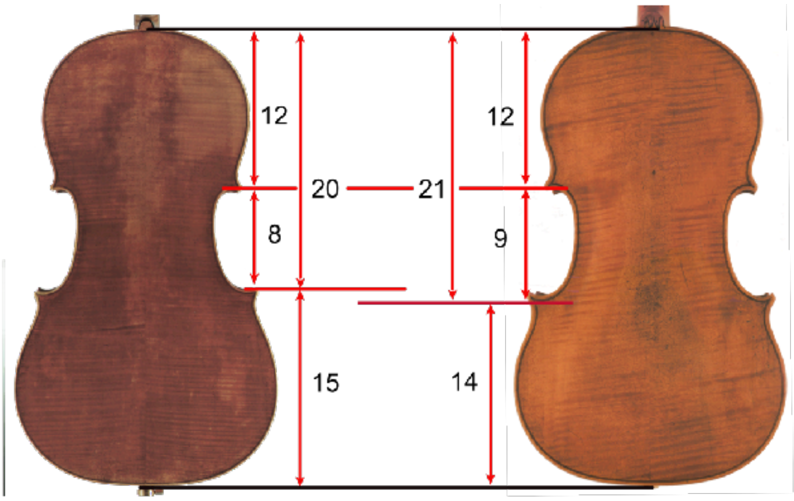


fig 20

Andrea Amati made two sizes of violins whose difference is 1/35 of the length. This interval corresponds to the modular value of the tone.

CONCLUSION

« ….with the exception of those investigating of the causality of things » Andrea Palladio (1508-1580)

All variations in measure cannot be reduced to intervals found in Greek harmony. Other ratios (5/7, 5/8, 7/4 ...) are frequent and I had the opportunity to demonstrate in the analysis of the lute of Arnault de Zwolle that the origin of these ratios is founded on an elaborate use of proportional sections (Traité de Lutherie-the violin and the art of measurement, François DENIS, aladfi Nice, 2006, pp 48-56).
Our purpose here is to emphasize that one of the keys to our inquiry and admiration for things from Antiquity is not a search for secrets and magical spells but for sensible experience, experience which precedes theory, having sensed the concept of 'harmony.
In his treatise I quattro libri dell'architettura Palladio states "The proportions of voices are harmony for the ears, those of measurements are harmony for the eyes. Such harmonies are often very pleasing without anyone knowing why, with the exception of those investigating of the causality of things. (Quoted by Wittkover note 46 p135)
With an invitation to exercise a sensitivity enlightened by the spirit, we have tried here to share the experience proclaimed by this great Italian architect.

.